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Are Common Core Standards for Math in Grades Three and Four Reasonable? Rethinking Word Problems Using a Letter for the Unknown

By Henry Borenson

The *Common Core State Standards in Mathematics* (CCSSM 2010), referred to as Standards here, set ambitious goals for third- and fourth-grade students. Not only are they expected to solve complex word problems—two-step problems at third grade (3.OA.8), multistep problems at fourth grade (4.OA.3)—but also to represent these problems with an equation using a letter for the unknown quantity. The latter is an advanced algebraic skill that usually is not introduced until later grades on international testing in the highest-achieving countries

For example, according to Ginsburg, Leinwald, and Decker (2009), it is not until the fifth grade that students in Hong Kong, Korea, and Singapore are expected to use a letter to represent the unknown. The example they provide is: "John is *x* years old now. How old will he be after 10 years?" (p. 33).

Even within the Standards, it is not until the sixth grade that students are specifically expected to represent an expression using a letter for the unknown (6.EE.2a). The wording of this Expression and Equations (EE) goal is,

> Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation, "Subtract *y* from 5" as $5 – y$. (p. 43)

Yet the ability to represent an expression with a letter for the unknown is a *prerequisite* skill to representing a word problem with an equation with a letter for the unknown —something that is expected in the Standards for third- and fourth-graders.

This presents a quandary: The Standards expect third- and fourth-grade students to use a skill that is not specifically taught, if the Standards are followed, until two or three years later in the sixth grade.

Clearly, this seems to be an error in the Standards. But this unsupported expectation in the earlier grades also may present an instructional opportunity.

What makes algebraic equations with letters difficult? According to Ginsburg, Leinwald, and Decker (2009), algebraic notation can be challenging to students because it differs from ordinary numerical notation. For example, whereas 52 is the sum of 50 and 2, $5x$ is not the sum of 50 and x. Furthermore, students may have trouble with the meaning of $4x + 5y$ because it appears as though they are being asked to add apples and oranges.

Lam (2002), discussing the elementary math curriculum in the high-performing country of Hong Kong, states, "Algebra occupies a very minor place in the primary curriculum, for the simple reason that *algebra involves a certain level of abstraction* which should be introduced at a later stage of development, in this case P5 [fifth grade] onwards" (p. 206, emphasis added).

Internal inconsistency between 3.OA.8/4.OA.3 and 6.EE.2a in the Standards is obvious. However, talented third- and fourth-graders can learn how to deal with this level of algebraic abstraction if the abstraction is presented in an understandable way. An effective instructional strategy begins with transforming the word problem into a concrete or pictorial equation and then uses that representation to construct an abstract equation using a letter for the unknown.

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The approach outlined here is not currently being used in Hong Kong but is consistent with the pedagogical approach of that country's schools. According to Lam, "Student learning is expected to progress from the concrete to the abstract" (p. 204). That is the essence of the strategy here.

This strategy also is philosophically consistent with the pedagogical approach used in Singapore, where, according to Cai and colleagues (2011), "The intent of using the 'model method' is to provide a smooth transition from working with the unknown in less abstract form to the more abstract use of letters in formal algebra in secondary school" (p. 33).

Transforming a word problem into a concrete or pictorial equation: The Standards do not provide an example of the type of multistep word problem it has in mind for 4.OA.3, so let's consider the following example:

> Tom buys two packs of football cards to add to the six cards a friend gave him. His mother then gives him three packs as a present. Now he has as many cards as Jane who owns two packs and 18 loose cards. If all the packs have the same number of cards, how many cards are in each pack?

Figure 1 shows how this word problem can be concretely represented using the approach of Hands-On Equations (Borenson 2010, Borenson 2009). The pawn represents the number of cards in a pack. A numbered cube represents a number of loose cards. The left side of the balance represents the total number of cards Tom has; the right side shows the number Jane has, with the loose cards represented by the sum of the three cubes. The balance image indicates that both sides have the same value. This equation using concrete objects fully represents the conditions of the problem.

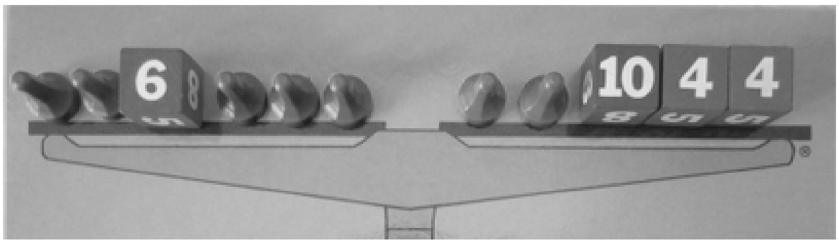

Figure 1. A concrete equation of the multistep word problem.

We can now assign a value to the pawn and designate it by a letter, namely x. The x represents the unknown number of cards in a single pack. The objects on the balance scale in Figure 1 are additive, as are weights on a scale, thus the concrete representation can be stated algebraically as $x + x + 6 + x + x + x = x + x + 18$ or $2x + 6 + 3x = 2x + 18$. Either of these equations would satisfy the requirement of goal 4.OA.3 in the Standards.

The Standards do not require fourth-grade students to solve the linear equation but simply to find the solution to the multistep word problem. Hands-On Equations students, however, would easily solve the concrete equation algebraically by physically removing, first, two pawns from each side and then a value of 6 from the number cubes on each side (Borenson 2011). Doing so leaves a reduced setup of three pawns on the left and a value of 12 in cubes on the right. Consequently, it is easy to see that each pawn has a value of 4, the number of cards in a pack. Counting 4 for the pawn in the reset physical representation shows that both sides have the same value, namely, 26.

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Just as students can represent the above problem concretely, they can also do so pictorially by drawing a picture of the balance scale and drawing shaded triangles and boxed numbers to represent the pawns and the numbered cubes, respectively. Arrows can be used to show the removal of pawns and cubes from each side of the balance.

A study conducted in the spring of 2008 with talented third-graders demonstrated that with appropriate instruction young talented students can make significant progress in pictorially representing and solving two-step and multistep word problems (Borenson 2009). For example, whereas 28 percent of the 195 students in the study solved the above multistep word problem correctly on a pre-test, 88 percent solved it correctly on the post-test. The instruction, using the Hands-On Equations strategy, involved seven lessons working with equations and another six working on representing word problems concretely or pictorially.

Conclusion: Goals 3.OA.8 and 4.OA.3 in the Standards, requiring a student to solve two-step and multistep word problems and to represent the problems using a letter for the unknown, appear to be misplaced, because a prerequisite skill involving a letter for the unknown is goal 6.EE.2. Nonetheless, talented third and fourth graders *can*—and have been shown to—meet the goals of 3.OA.8 and 4.OA.3, depending on the complexity of the problem. These young students first represent the word problem in a concrete or pictorial equation and then transform that representation into an abstract equation using a letter for the unknown. They then solve the problem algebraically from the concrete or pictorial representation. By using this approach, proceeding from the concrete to the abstract, U.S. students can exceed their age/grade counterparts in high-achieving countries on this goal.

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