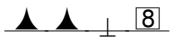
Teaching Points/Objectives by Lessons
Hands-On-Equations®

Equation	Teaching Point/Objective
Level I, Lessons 1 – 7	
Lesson 1 	In any specific problem, all the blue pawns have the same value, and the scale is in balance when both sides have the same value. Solve for the pawn using *trial and error* and intuitive thinking.
Lesson 2 $2x + x = x + 8$	The pawn has a special name, "*x*". Transform the equation into its physical representation, using the blue pawn for the *x* and the number cube for the number constant. The two sides of the equal sign become the two sides of the scale. Use trial and error to find the value of *x*.
Lesson 3 $4x + 2 = 3x + 9$	The *legal move* with pawns is introduced: we may remove the same number of blue pawns from both sides of a balanced system (Subtraction Property of Equality).
Lesson 4 $4x + 5 = 2x + 13$	Subtract the same number value from both sides of a balanced system (Subtraction Property of Equality).
Lesson 5 $5x - 3x + 2 = x + 5$	Students take away pawns as part of the setup process: distinguishing the set up from a legal move, which comes <u>after</u> the setup has been completed.
Lesson 6 $2(x + 3) = x + 8$	The students learn that the number outside the parenthesis tells us how many times the expression inside the parenthesis is set up on the balance scale. (Some students learn the distributive law without being taught!)
Lesson 7 $4x + 3 = 3x + 9$	Transfer the hands-on experiences of Level I to a pictorial system. The *x*'s are represented by shaded triangles. The number constants by boxed numbers. The scale by a picture of the scale. No plus signs are placed on the scale, only pawns or cubes.

# Level II, Lessons 8 – 16	
Lesson 8 $2(3\bigstar + 1) = 2\bigstar + 10$	The white pawn is given the name of "star" and it is written as an *x* with a bar through it. Students learn that the same legal moves that were performed with the blue pawn may also be performed with the side pawn.
Lesson 9 If $x = 2$, evaluate: $2x + \bigstar + x + \bigstar + 5$	The students learn to distinguish an expression from an equation (two expressions joined by an equal sign), and they learn to evaluate expressions. The students are informed that *x* and star are opposites of each other, and hence their sum is zero ($x + \bigstar = 0$). [Property of Additive Inverses]
Lesson 10 $3x + \bigstar = x + 4$	The students learn a new legal move: a blue and a white pawn may be removed from the <u>same side</u> of setup. <u>One hand</u> is used in carrying out this legal move [Addition Property of zero]
Lesson 11 $2x = \bigstar + 6$	New legal move: We may add the same element to both sides. [Addition Property of Equality]
Lesson 12 $2x = \bigstar + 6$	New legal move: We may add a blue and a white pawn to the <u>same</u> side of a setup. We use one hand to do so. [Addition Property of Zero]
Lesson 13 $2x + \bigstar + 3 = 2\bigstar + 15$	Simplify the setup so that number cubes are on only one side of the setup; then work with the pawns.
Lesson 14 $2x - \bigstar = 12$	In order to subtract an element (blue or white pawn) not already on the balance scale, we add a *convenient zero* in the form of a blue pawn and a white pawn <u>as part of the setup process</u>, and then subtract the element as part of the setup process.
Lesson 15 $2x - (-x) + 4 = 2(-x) + x$	Students learn to recognize the symbol $(-x)$ as another name for star. The symbols "$(-x)$" is read as *the opposite of x.* Students set up $(-x)$ by using the white pawn, just as they would in setting up star.
Lesson 16 $2x = \bigstar + 6$	The hands-on experience of Level II is transferred to a pictorial system using only paper and pencil: the white pawns are shown as triangles which are <u>not</u> shaded in, so as to distinguish it from the blue pawn, which is shaded in.

Level III, Lessons 17 – 25	
Lesson 17 (-3) + (+5) = +2 (+5) – (-3) = +8	The students are introduced to the green cube as having a negative value. Hence, a red 2 cube and a green 2 cube, together, have a value of zero. To add integers, students rename the cube with the larger number so that one of the addends matches the cube with the smaller number, and together, these have a sum of zero. The other addend gives the sum. Students subtract integers by first adding a convenient zero in the form of the subtrahend and its opposite. They then remove the subtrahend and end up adding the opposite of the subtrahend to the original number (the minuend).
Lesson 18 $x + 3 = -5$	The same legal moves performed with the pawns may be performed with the cubes, e.g., we may add (or subtract) the same value and color numbered cube from sides of the setup. The goal is to isolate the variable (the pawn) by removing the number constant next to it.
Lesson 19 $\cancel{x} - 3 = -6$	Add a *convenient zero* in the form of a red cube and a green cube during the setup of the equation in order to subtract an element not there originally.
Lesson 20 $2x + (-3) = 5 + x$	Solve equations using red and green cubes as well as pawns of one color.
Lesson 21 $2\cancel{x} + 3 = -6 + x$	Solve equations using red and green cubes and blue and white pawns.
Lesson 22 $2(x - 1) + 4\cancel{-x} = x + (-8)$	Solve equations containing multiples of parenthetical expressions involving the green cube.
Lesson 23 $x - 2(\cancel{x} + 2) = 5$	Subtracting a multiple of a parenthetical expression.
Lesson 24 $x - 2(-x) - 3 = 5 + (-x)$	Use blue and white pawns and the red and green cubes to solve equations with the notation of "$(-x)$".
Lesson 25 $2\cancel{x} + 3 = -6 + 3$	The concrete experiences of Level III are transferred to a pictorial system involving only paper and pencil. The negative number are represented by circled numbers

Note; Lessons 18 - 20, and 22-24 are NOT included in the Day1 Making Algebra Child's Play workshop.