

LESSON	OBJECTIVES	NCTM MATH STANDARDS: GRADES 3 – 5
Hands-On Equations(R) Learning System: Level I		NCTM Number and Operations Standard; Algebra; Problem Solving; Communication; Representation
Lesson 1	Students will use a symbol to represent an unknown.	Represent the idea of a variable as an unknown quantity using a letter or a symbol.
	Students will demonstrate an understanding of the concepts of <i>equivalence, variable, constant, equation, equal to (=), and not equal to (≠)</i> .	Represent the idea of a variable as an unknown quantity using a letter or a symbol.
	Students will use substitution and mental math to find and check solutions to physical or pictorial representations of algebraic equations in one variable.	Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method of tools.
		Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.
	Select and use various types of reasoning and methods of proof.	

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Lesson 2	Students will demonstrate an understanding of the abstract representation of an algebraic equation in one variable.	Create and use representations to organize, record, and communicate mathematical ideas.
	Students will model an abstract algebraic equation using concrete materials.	Create and use representations to organize, record, and communicate mathematical ideas. Select and use various types of reasoning and methods of proof.
	Students will use repeated addition to represent multiplication in an algebraic expression.	Understand various meanings of multiplication and division.

Lesson 2 Cont'd.	<p>Students will demonstrate an understanding of the concepts of <i>equivalence</i>, <i>variable</i>, <i>constant</i>, <i>equation</i>, <i>equal to</i> (=), and <i>not equal to</i> (\neq).</p>	<p>Represent the idea of a variable as an unknown quantity using a letter or a symbol.</p>
	<p>Students will use concrete models, guess-and-check, and number sense to solve one- and multi-step algebraic equations in one variable.</p>	<p>Apply and adapt a variety of appropriate strategies to solve problems.</p> <p>Select, apply, and translate among mathematical representations to solve problems.</p> <p>Create and use representations to organize, record, and communicate mathematical ideas.</p>
	<p>Students will use substitution and mental math to check solutions to algebraic equations in one variable.</p>	<p>Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method of tools.</p> <p>Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.</p> <p>Select and use various types of reasoning and methods of proof.</p>

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Lesson 3	Students will use concrete models to represent algebraic equations in one variable with unknowns on both sides of the equation.	<p>Create and use representations to organize, record, and communicate mathematical ideas.</p> <p>Select and use various types of reasoning and methods of proof.</p>	
	Students will use kinesthetic motions (physical actions) to solve algebraic equations in one variable with unknowns on both sides of the equation.	<p>Apply and adapt a variety of appropriate strategies to solve problems.</p> <p>Select and use various types of reasoning and methods of proof.</p>	
	Students will demonstrate an understanding of the concepts of <i>equivalence, variable, constant, equation, equal to (=), and not equal to (\neq)</i> .	Represent the idea of a variable as an unknown quantity using a letter or a symbol.	
	Students will use repeated addition to represent multiplication in an algebraic expression.	<p>Understand various meanings of multiplication and division.</p> <p>Understand the effects of multiplying and dividing whole numbers.</p>	
	Students will use the Subtraction Property of Equality with variables to form equivalent statements when solving algebraic equations in one variable with unknowns on both sides of the equation.	<p>Develop and evaluate mathematical arguments and proofs.</p> <p>Identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems.</p>	
	Students will use substitution and mental math to check solutions to algebraic equations in one variable.	<p>Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method of tools.</p> <p>Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.</p> <p>Select and use various types of reasoning and methods of proof.</p> <p>Organize and consolidate their mathematical thinking through communication.</p> <p>Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.</p> <p>Analyze and evaluate the mathematical thinking and strategies of others.</p> <p>Use the language of mathematics to express mathematical ideas precisely.</p>	
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		GRADES 3 - 5
Lesson 4	Students will use concrete models to represent algebraic equations in one variable with unknowns on both sides of the equation.	<p>Create and use representations to organize, record, and communicate mathematical ideas.</p> <p>Select and use various types of reasoning and methods of proof.</p>
	Students will use kinesthetic motions (physical actions) to solve algebraic equations in one variable with unknowns on both sides of the equation.	<p>Select and use various types of reasoning and methods of proof.</p> <p>Apply and adapt a variety of appropriate strategies to solve problems.</p>
	Students will use the Subtraction Property of Equality with variables and with constants to form equivalent statements when solving algebraic equations in one variable.	<p>Identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems.</p> <p>Develop and evaluate mathematical arguments and proofs.</p>
	Students will use substitution and mental math to check solutions to algebraic equations in one variable.	<p>Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method of tools.</p> <p>Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.</p> <p>Select and use various types of reasoning and methods of proof.</p> <p>Organize and consolidate their mathematical thinking through communication.</p> <p>Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.</p> <p>Analyze and evaluate the mathematical thinking and strategies of others.</p> <p>Use the language of mathematics to express mathematical ideas precisely.</p>
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		GRADES 3 - 5
Lesson 5	Students will use concrete models and kinesthetic motions (physical actions) to represent and solve algebraic equations in one variable with unknowns on both sides of the equation.	<p>Create and use representations to organize, record, and communicate mathematical ideas.</p> <p>Apply and adapt a variety of appropriate strategies to solve problems.</p> <p>Select and use various types of reasoning and methods of proof.</p>
	Students will add and subtract monomials and combine like terms to form equivalent expressions when solving algebraic equations in one variable.	
	Students will use Properties of Equality, such as the Subtraction Property of Equality, to form equivalent statements when solving algebraic equations in one variable with unknowns on both sides of the equation.	<p>Identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems.</p> <p>Develop and evaluate mathematical arguments and proof.</p>
	Students will use substitution and mental math to check solutions to algebraic equations in one variable.	<p>Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method of tools.</p> <p>Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.</p> <p>Select and use various types of reasoning and methods of proof.</p> <p>Organize and consolidate their mathematical thinking through communication.</p> <p>Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.</p> <p>Analyze and evaluate the mathematical thinking and strategies of others.</p> <p>Use the language of mathematics to express mathematical ideas precisely.</p>
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		NCTM MATH STANDARDS: GRADES 3 - 5	
Lesson 6	Students will use concrete models to represent the multiplication of a binomial by a positive integer constant, such as $2(x + 1)$.	Create and use representations to organize, record, and communicate mathematical ideas. Understand various meanings of multiplication and division.	
	Students will use concrete models and kinesthetic motions (physical actions) to represent and solve algebraic equations in one variable with unknowns on both sides of the equation.	Create and use representations to organize, record, and communicate mathematical ideas. Select, apply, and translate among mathematical representations to solve problems. Apply and adapt a variety of appropriate strategies to solve problems. Select and use various types of reasoning and methods of proof.	
	Students will use algebraic properties, such as the Subtraction Property of Equality and the Distributive Property of Multiplication over Addition, to form equivalent statements and expressions when solving algebraic equations in one variable.	Understand and use properties of operations, such as the distributive of multiplication over addition. Identify such properties as commutative, associative, and distributive and use them to compute with whole numbers. Develop and evaluate mathematical arguments and proof.	
	Students will use substitution and mental math to check solutions to algebraic equations in one variable.	Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method of tools. Develop fluency in adding, subtracting, multiplying, and dividing whole numbers. Select and use various types of reasoning and methods of proof. Organize and consolidate their mathematical thinking through communication. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others. Analyze and evaluate the mathematical thinking and strategies of others. Use the language of mathematics to express mathematical ideas precisely.	
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Lesson 7	Students will use pictorial models to represent abstract algebraic equations, including equations that contain multiplication of a binomial by a positive integer constant, such as $2(x + 1)$.	Create and use representations to organize, record, and communicate mathematical ideas.
		Understand various meanings of multiplication and division.
		Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method of tools.
	Students will use pictorial representations to solve algebraic equations in one variable with unknowns on both sides of the equation.	Apply and adapt a variety of appropriate strategies to solve problems.
		Select, apply, and translate among mathematical representations to solve problems.
		Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method of tools.
		Select and use various types of reasoning and methods of proof.
	Students will use algebraic properties, such as the Subtraction Property of Equality and the Distributive Property of Multiplication over Addition, to form equivalent statements and expressions when solving algebraic equations in one variable.	Identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems.
		Understand and use properties of operations, such as the distributive of multiplication over addition.
		Identify such properties as commutative, associative, and distributive and use them to compute with whole numbers.
		Develop and evaluate mathematical arguments and proof.
	Students will use substitution and mental math to check solutions to algebraic equations in one variable.	Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method of tools.
Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.		
Select and use various types of reasoning and methods of proof.		
Organize and consolidate their mathematical thinking through communication.		
Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.		

		Analyze and evaluate the mathematical thinking and strategies of others.
		Use the language of mathematics to express mathematical ideas precisely.
LESSON	OBJECTIVES	NCTM MATH STANDARDS: GRADES 3 – 5
	Hands-On Equations(R) Verbal Problems Book: Introduction & Level I	NCTM Number and Operations Standard; Algebra; Problem Solving; Communication; Representation
Level I Verbal Problems	Students will analyze verbal problems and determine what strategy is best for representing the problem using concrete models or pictorial symbols.	Apply and adapt a variety of appropriate strategies to solve problems.
		Use representations to model and interpret physical, social, and mathematical phenomena.
		Recognize reasoning and proof as fundamental aspects of mathematics.
		Organize and consolidate their mathematical thinking through communication.
	Students will use mathematical models (e.g., concrete, pictorial) to represent and solve application problems involving quantitative relationships with rational numbers.	Create and use representations to organize, record, and communicate mathematical ideas.
		Select, apply, and translate among mathematical representations to solve problems.
		Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.
		Apply and adapt a variety of appropriate strategies to solve problems.
		Solve problems that arise in mathematics and in other contexts.
	Students will write down the representation of the unknown elements of the verbal problem.	Represent the idea of a variable as an unknown quantity using a letter or a symbol.
	Organize and consolidate their mathematical thinking through communication.	
	Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.	
	Create and use representations to organize, record, and communicate mathematical ideas.	
Students will use multiple strategies (e.g., a 5-step problem	Apply and adapt a variety of appropriate	

Level I Verbal Problems Cont'd.	solving process, breaking a problem into parts, and determining whether a problem has too much information or not enough information) to solve real-world application problems.	strategies to solve problems.
	Students will represent physically or pictorially given relations, such as Jim's age 10 years from now given his current age, the amount of rain that falls in three months given the amount it falls in one month, the distance travelled by a train in two hours given the distance it travels in one hour, or a number that is $\frac{2}{3}$ of another number.	Create and use representations to organize, record, and communicate mathematical ideas.
		Use representations to model and interpret physical, social, and mathematical phenomena.
		Build new mathematical knowledge through problem solving.
		Express mathematical relationships using equations.
		Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.
		Recognize reasoning and proof as fundamental aspects of mathematics.
		Select and use various types of reasoning and methods of proof.
	Students will represent and solve application problems including problems involving distance, money, age, and patterns of numbers (e.g., consecutive numbers, consecutive even numbers, and consecutive odd numbers).	Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.
		Create and use representations to organize, record, and communicate mathematical ideas.
		Select, apply, and translate among mathematical representations to solve problems.
		Use representations to model and interpret physical, social, and mathematical phenomena.
		Identify and describe situations with constant or varying rates of change and compare them.
Solve problems that arise in mathematics and in other contexts.		
Recognize reasoning and proof as fundamental aspects of mathematics.		
Organize and consolidate their mathematical thinking through communication.		
Students will provide the answer to verbal problems in full sentence format.	Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.	
	Use the language of mathematics to express mathematical ideas precisely.	
	Organize and consolidate their mathematical thinking through communication.	
	Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.	

Students will use substitution and mental math to check solutions to application problems involving algebraic equations in one variable.

Use the language of mathematics to express mathematical ideas precisely.

Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method of tools.

Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.

Select and use various types of reasoning and methods of proof.

Organize and consolidate their mathematical thinking through communication.

Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Analyze and evaluate the mathematical thinking and strategies of others.

Use the language of mathematics to express mathematical ideas precisely.