Teaching Pre-Algebra to Seventh Grade Resource Room Students Using
The Hands-On Equations Learning System

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ABSTRACT

Teaching Pre-Algebra to Seventh Grade Resource Room Students Using
The Hands-On Equations Learning System

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Gratz College, 2003
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This study examined the effects a manipulative hands-on approach to understanding variables and pre-algebra had on a small group of seventh grade classified students with learning differences. The purpose of the study was to research the commercial Hands-On Equations System, created by Dr. Borenson, which is meant for the fifth grade through adult age groups. It is ideal for learning different students, using a visual, kinesthetic, and game-like approach that provides a strong foundation for later algebraic studies. Data was collected using triangulation, and included student surveys, student/teacher interviews, student journal writing, pre-testing, post-testing, and a teacher’s daily log for observations. Results showed all five students with at least 80% mastery upon completion of Level I of this program. Math attitudes were changed for the better. Students also displayed improved self-confidence and self-esteem in math class using this system.
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Chapter 1

Introduction

Searching For a Better Way

Teaching simple linear algebraic equations is now one of the essential elements in the mathematics curriculum for seventh grade in my school district. Many students this age are still primarily concrete learners so this abstract concept is frustrating and can be viewed as threatening. Students with learning differences compound the resistance to these new concepts. In my quest to find a better way to teach seventh grade classified resource room students pre-algebra, I am focusing this teacher action research study on a program called, Hands-On Equations, by Dr. Henry Borenson. He claims this manipulative system will change abstract linear equations into concrete equations, and that students are quickly successful using this system, which increases confidence and attitude about pre-algebra (Borenson, 1994). I will be assessing the impact this program makes on my small but diverse group of classified students.

In order to more fully document the story of my teacher action research, I would like to present my personal history and school district background. I will then explain my study and investigative plan.

Personal History

Life is a journey of many paths.

As I reflected on how I became a special education teacher many paths emerged.
During my youth, school was not my favorite thing. I struggled all through grammar school due to weak reading skills. This impacted most subjects in school except mathematics, which quickly became my favorite. Math allowed the platform to think about the problem, visualize it, and solve it. I did not have to rely on reading comprehension, memorization, or recall, which were areas of weakness for me. Luckily, by the time I got to high school, I had improved my reading and had developed many techniques to help me be successful in all subject areas. This was a path that prepared me for understanding and teaching learning different children with my many self taught techniques.

Taking care of seven children left little time for my parents to sit and help with school studies. My youngest sibling was in fourth grade when she was classified with Dyslexia. As my parents discussed this at home, an older sibling added that he had similar visual conditions. He thought everyone saw things the same way as he. He had stayed back in school and barely graduated from high school as he struggled without the benefit of knowing he too had a learning disability. Growing up watching my siblings struggle academically and emotionally greatly influenced my desire to help children with learning differences.

I am reminded of the final display in the videotape, The FAT City Workshop by Richard Lavoie (1989), that states, “Learning Disability- The real challenge is educating those who don’t have one.” I can associate with this as a result of my own temporary struggles and those of my siblings. This was another pathway toward becoming a teacher.
When I was twelve years old an event unfolded which took me down a different path. A child in town needed volunteers to do Delecato Patterning, a method of physically going through the early steps of motor development, like crawling, to retrain the brain. I felt a calling to participate and thus began my interest in special education. I stayed involved with this family for many years as a baby-sitter and friend. Little did I know that in the future this family would be instrumental in acquiring my first teaching position.

One more path that influenced my decision to become a teacher was the high regard and value my family attached to a college education. Many of my siblings went on to become professionals in medicine, biology, horticulture, and investments. I became the first female in my family to graduate from college, as well as the first teacher.

I graduated from William Paterson University in 1974 with degrees in Special Education, Elementary Education, and Pre-School Handicapped. I taught in a special education self-contained setting for a year. The following three years I taught at a private school instructing mathematics to sixth, seventh, and eighth grade students. With the arrival of my first child, I became an at-home mom.

During the precious years of raising my two children, I volunteered to run programs for the church, community center, library, a national woman’s club, and the PTA at school. My interest and focus was always centered on children.

I returned to teaching at a local public school eleven years ago. I have learned from, and enjoyed, all the paths that have led me on the road to Little Falls Middle School. Here I teach mathematics to seventh and eighth grade classified students in both a pullout resource room setting, and as inclusion support for the mainstreamed students. I
love teaching what I teach, whom I teach, and the school environment I am in. I enjoy going to a setting where student and teacher both grow in the many areas of life.

**Little Falls Middle School**

Little Falls has long enjoyed a reputation as a small and friendly place, with a population of about 11,200 people. The town is 2.5 square miles and is located approximately 17 miles west of New York City. It is predominantly white, 88.4%, Hispanic, 5.3%, and Asian, 4.2% (Census, 2000). The socioeconomic level is middle class.

Little Falls Middle School has a population of 479 students and provides programs for children in grades five through eight. Average class size is 24.2 students, with 98% of the school population speaking English, and 2% are limited English proficient. The student mobility rate is 4%. Student attendance rate is 95.7%, and the faculty attendance rate is 96.3%. The length of the school day is 6 hours: 35 minutes. *The Little Falls School Report Card 2000-2001* states the focus of the school's programs is centered on the developmental characteristics of emergent adolescent learners.

Students spend the majority of their day in heterogeneous settings with provisions made to meet individual student needs. The operational philosophy emphasizes personalized education in which the school is modeled on a family where caring adults guide and instruct young people. *The Commissioner of Education recommended Little Falls School District to be fully certified, which indicated the district has earned the state’s highest approved rating. The Student Council has been selected many times as a Gold Honor Council. This award recognizes accomplishments in community service, and school*
citizenship. In December of 2000, the Township overwhelmingly approved the passing of a bond referendum allowing the district to completely renovate and re-open an unused town school building to alleviate over-crowded school conditions. All those involved at Little Falls Middle School realize that children only reach their potential when parents and educators work cooperatively (Little Falls Schools, 2002).

Rationale

Teacher action research.

In, *Action Research: An Educational Leader's Guide to School Improvement*, Glanz (1998) sums up action research as a type of applied research that is conducted by teachers to improve practices in educational settings. Action research, Glanz noted, utilizes an array of methodologies, and is generally identified with qualitative approaches.

I believe that teachers are educational experts who possess special knowledge and skills allowing them to influence instruction for their students in positive ways. Doing research can be thought of as investigating or a process of gathering information. Teachers routinely gather information to assess students' achievements. Some of the assessments are more traditional. Many times a teacher will observe and just know intuitively, but this alone is not a valid measure. According to Glanz, research is a more disciplined systematic approach to study a given educational problem. Research should not be reserved for those with doctoral degrees or scientists. Research is something all teachers can do to help understand, if not solve, some of the urgent issues confronted on a
daily basis in schools. To do so thoughtfully, comprehensively, and intelligently is what teacher action research is all about (Glanz, 1998).

Glanz continued to say that action research is cyclical, meaning the process does not have to stop at any particular point. First, the focus of the research is selected. Then data is collected, analyzed, and an action is taken. Information gained from research may open new paths of research. Often when reflection is done on the results of a research, modifications are made and the process continues onward (Glanz, 1998).

As a teacher I am constantly involved in assessing instruction and seeking ways of improving the educational process. As noted by Glanz, teacher action research is systematic, intentional, and ordered. It empowers those who participates in the process, creates a more positive school climate, promotes reflection and self-assessment, enhances decision making, and instills a commitment to continuous improvement. I am looking forward to using teacher action research, which will afford me the opportunity and tools necessary to accomplish the research for this thesis.

The Study

Why try a new program?

Algebra is considered the gatekeeper to higher education. Being successful in algebra usually leads to students taking more difficult mathematics courses, and higher cognitive skill courses. With algebra being so important to a student’s future, one must wonder why there is such a low student success rate in algebra? Some believe the adolescent student’s cognitive development may not be ready for the abstract concepts in algebra (Pallard, 1979). Other researchers believe children as early as third grade can be
taught algebraic concepts if the concepts are developed using concrete materials (Borenson, 1987; Thompson, 1988). Usiskin (1987) cited data from Japan that indicated algebra could be successfully taught to eighth-graders. He insisted that not only could algebra be taught at an earlier age, but that it must be.

Teaching pre-algebra in seventh grade is apart of my district’s math curriculum. Starting with pre-algebra at this age is in response to recommendations made by the National Council of Teachers of Mathematics, upon which the New Jersey Core Curriculum Standards for mathematics are based. In 1989, the National Council of Teachers of Mathematics developed curriculum and evaluation standards that outlined important mathematical topics for grades K-12. The concept of variable was one of the major algebraic concepts to be taught to students in grades 5-8. Lowering the instructional grade level for this concept was to answer a call for better-prepared mathematics students and to increase the success rate for algebra.

As a pre-algebra math teacher I have concerns about seventh grade classified math students who must be pulled-out to a resource room setting because they are unable to be mainstreamed for various reasons. These students feel they are math failures and come to class with math anxiety. Pre-algebra and using linear equations with variables can cause panic among my classified math students. They experience limited success and are often frustrated with pre-algebra. To me, the problem is that these equations are too abstract and most of my students are concrete learners. Though I have used a variety of methods, including many of my own developed techniques, when it comes to making pre-algebra concrete, I have never been fully satisfied with the results. Attending a
workshop about the *Hands-On Equations Learning System* made me wonder if this could be the system that my students would find meaningful and interesting.

Borenson (1998), the developer of this system, claims the visual and kinesthetic aspects of this program make it ideal for use with learning different students from fifth grade through college. He also stated that the essential algebraic concepts will be developed, which will provide a strong foundation for later algebraic studies.

If students can learn these concepts in the middle-grades using concrete manipulatives, then algebra will be easier in the future. Since having fun and experiencing success early with variables and linear equations may be just what my students need for their self-confidence and their future math classes, I decided to try this program.

**The Research Question**

The purpose of this study is to look at the effects a manipulative hands-on approach to understanding variables and pre-algebra has on a small group of seventh grade classified students with learning differences. My research inquiry will be focused on what impact the commercial *Hands-On Equation System* makes on seventh grade classified resource room math students. By the end of the seventh lesson, will these students be able to physically set up and solve given equations? How else will the students be affected by using this system? Will there be a change in attitude about mathematics by the end of the seven lessons? Various forms of cumulative assessment will be used such as pre-testing, interview, journal writing, student/teacher conference, teacher's daily log of observations, and post-test.
Overview

In Chapter 1, I have introduced my research topic of searching for a better way to educate students, along with a personal history of the many paths that have lead me to the road of teaching classified students mathematics. I then continued with my rationale for a teacher action research study, which is about using manipulatives to teach pre-algebra. The purpose of the study is to assess if the commercial Hands-On Equation System does what it claims; the research examines how this system will impact upon the students.

Chapter 2, I will provide definitions of terms used in this thesis, and review the literature on the topic of mathematics such as the importance of math, a brief history of algebra, and opposing views of instructional approaches to algebra. The literature review will continue with curriculum and evaluation standards set by the National Council of Teachers of Mathematics (NCTM), cognitive developmental stages, use of manipulatives, and the Hands-On Equation system. Finally, the literature research will look at the special needs learner including laws, least restrictive environment, and the attitudes of these special learners towards mathematics.

In Chapter 3, I will provide the details of the methods used to collect, organize, and validate the data in conducting the research. This data will be collected at Little Falls Middle School, in Little Falls, New Jersey during the fall semester of the 2002-2003 school year.

In Chapter 4, I will report the results of my action research. I will explain how the triangular approach to data collection supports the specific findings.

In Chapter 5, I will summarize, discuss, and reflect on the results.
Chapter 2

Review of the Literature

Overview

The purpose of this study is to look at the effects a hands-on approach to understanding the concepts of algebra has on a small group of seventh grade classified students with learning differences. There are many factors to be considered when choosing a new technique to introduce the concepts of algebra. Some of them will be examined in the following review of literature.

Introduction

This chapter begins with definitions and includes literary research on the importance of studying mathematics, as well as a brief history of algebra. It continues with a discussion of opposing views about algebra and how those views affect the instructional approach that is used for teaching this subject. The literature review also investigates the standards for learning algebra set by the highly recognized National Council of Teachers of Mathematics (NCTM). This will be followed with some issues and trends used in the teaching and learning of algebraic concepts in the middle school. Topics about cognitive developmental stages, the use of manipulatives in mathematics and algebra, and an approach to teaching algebra using the Hands-On Equation System is examined. To view this information in light of the classified students, mention will be made of the history of present laws, the least restrictive environment, classification, and attitudes of the learning different student towards mathematics.
Definitions

(1) Classified- used interchangeably with learning different, special needs, handicapped, and learning disability- this refers to children whose educational and social needs cannot be met solely within the general education classroom, and require special education and related services (that is counseling, educational instruction speech, language, physical therapy, etc.). It is necessary that the students have an Individual Educational Plan (IEP) to provide an appropriate program for the child’s education (Vaughn, Bos & Schumm, 2000, p. 9).

(2) Algebra- a generalization of arithmetic in which letters representing numbers are combined according to the rules of arithmetic, ... logic concerned with the properties and relationships of abstract entities manipulated in symbolic form (Webster, 1994, p. 28).

(3) Middle Grade Students- students from grade 5 through 8.

(4) Concrete manipulatives- objects, which represent mathematical ideas that can be abstracted through physical involvement with the objects (Young, 1983, p. 12).

(5) Variable- a quantity that may assume any one of a set of values, a value represented by a symbol like X (Webster, 1994, p. 1306).

The Importance of Studying Mathematics

Many may question why mathematics is important. Rick Norwood (2000), a professor at East Tennessee State University, who teaches the history of mathematics, answers this question when he states, “Mathematics is important to many jobs, many
problems can only be solved by using mathematics, and we need to learn mathematics to understand science. The most important reason for learning mathematics, not just by rote but with real understanding, is that it trains us to think clearly and arrive at answers not just for today, but for all time”. This is the mission at hand when teaching any child mathematics, to be a problem solver in the outside world around us (p. 253).

Mathematics, and science, is often considered a worldwide language because it is the same in all cultures. Norwood stated, “Mathematics is truth that has stood the test of time from ancient civilizations, it is the same in every culture, and different methods give the same answer” (p.252).

Yet the middle school children in the United States appear to be falling behind. William Schmidt, who helped coordinate the Third International Mathematics and Science Study (TIMSS) stated:

TIMSS is the most extensive and far-reaching comparative study of mathematics and science ever attempted. Of the findings Schmidt stated, “The results indicate that U.S. mathematics education in the middle grades is particularly troubled”. While the 4th graders scored somewhat above average, U.S. 8th graders scored below it, a clear signal that the problem with U.S. middle school math is not the students but the system. Problems with the present system are low expectations for student achievement; insufficient professional development; and shallow, repetitive curriculums (Mann, 2000, p.1).

By understanding Schmidt’s findings, educators are not at a loss, but given a powerful insight to correct this situation in the schools. Glenda Lappan, president of the National Council of Teachers of Mathematics (NCTM) responded to the situation with
optimism by stating, "Educators should make sense of what TIMSS tells us so that we can make a difference in what we do in the classroom". Because TIMSS includes information about those systems in which student achievement is high, the findings are very helpful. There is also a state-by-state break down. Some parts of the U.S. did very well, and those systems should be examined to understand what methods were used to teach mathematics. (Mann, 2000, p.2).

There is clearly no doubt that mathematics is important to the education of a child. Lynn Steen, who wrote the foreword for Sheila Tobias' book, *Succeed With Math* (1987), makes the point why we need to correct the shortcomings in the teaching techniques used in mathematics. She stated:

Despite the pervasive nature of mathematics as the power broker in so many important arenas of human activities, it is largely hidden from public view. Those who fear it don’t want to think about it; those who escaped from school without it don’t want to talk about it; and those who practice it don’t want to discuss it in public (p. xvii). ... The paradox of our times is that as mathematics becomes increasingly powerful, only the powerful seem to benefit from it. The ability to think mathematically -- broadly interpreted -- is absolutely crucial to advancement in virtually every career. The well being of our nation depends on the ability of our youth to succeed with mathematics. For this to happen we must make mathematics visible by destroying myths, overcoming anxieties, and removing barriers (Steen, 1987, p. xviii).
The History of Algebra

Algebra has been the pursuit of intellectuals for centuries. Even the great Greek philosophers and other great civilizations recognized its powerful value. This brief account of the history of algebra was found in the book, The History of Mathematics, written by David Burton (1991):

Although a number of historians claim that the Egyptians or perhaps the Babylonians were responsible for the advent of algebra, more people attribute its first appearance to the Greeks. Diophantus (c.275), often associated with Diophantine equations, is sometimes called the father of algebra. However, the most popular opinion is that the subject of algebra was first treated in a formal way by the Arabs, in particular, by Muhammed ibn M us a al-Khow arizm i (c.820), who named the subject. It is interesting that the Arabic word jabr means, "setting bone." Al-jabr actually means "science of reunion and reduction," where reunion refers to the "transference of negative terms from one side of the equation to the other" and reduction in "combination of like terms on the same side or the cancellation of like terms on opposite sides." Al-jabr translates as "the reunion of broken parts" as illustrated by rewriting 3x = 6-2x as 5x = 6. (p.174).

Algebra for everyone.

"Algebra for all" is a frequently expressed goal in the literature on school mathematics reform. In today's educated society algebra's value is recognized as essential to a well-rounded education and one's future. Many states have established algebra as a graduation requirement. National organizations have been created to further the academic education of U.S. citizens. The College Board's Equity 2000 project calls
for the elimination of tracking and the implementation of a college preparatory curriculum for all, which includes algebra.

Willis (1993) stated, "In discussions of the mathematics achievement—or lack thereof—of U.S. students, algebra is often the focus of attention. The reason is simple: Algebra is so central to mathematics that if students do not learn it, their future options are severely limited". Mary Lindquist, president of the NCTM adds, "Without it, students are cut off from future mathematics study and from careers in technology and the sciences. Algebra has been identified as a 'gatekeeper' not only for math but for science and other higher learning" (p. 3).

The National Council of Teachers of Mathematics has been established to realize the goal of algebra for all and teaching it in a meaningful way. The following recommendations were made by Jim Kaput, chairman of the Working Group on Learning/Teaching of Algebra and Quantitative Analysis of the National Center for Research in Mathematical Sciences Education in association with the NCTM. (Abridged here from NCTM 1993):

1. Algebra must be part of a larger curriculum that begins at an early age.

2. The algebra curriculum should be organized around the concept of patterns.

3. New modes of representation need to complement the traditional numerical and symbolical forms.

4. Algebraic thinking, which embodies the construction of patterns, generalization, active exploration, and conjecture, must be reflected throughout the curriculum across many grade levels (Chambers, 1994, p.85).
Lynn Steen, the executive director of the Mathematics Science Education Board, appears to have a cautious view about the ‘algebra for all’ system. Though she is a strong proponent of school mathematics reform, Steen warns against ‘algebra for all’ if that algebra is taught in the traditional memorization and manipulation of symbols. Steen stated, “Let’s face it, for most students the current school approach to algebra is a disaster” (Chambers, 1994, p.85).

Two Views of Algebra in the Classroom

Algebra, just like many other subjects, can be taught with a variety of methods and theories. Research studies combine all these techniques into two basic categories, which determines how and when algebra is taught. Davis (1985) reported:

The first view was one of doing mathematics where “algebra” was the meaningless, rote manipulation of symbols. The second view of learning mathematics was that which students do to build up mental representations for the key ideas of algebra. Viewing algebra as the rote manipulation of symbols was what has kept it from being taught earlier. . . . Teachers in the United States have been under great pressure to teach a tremendous amount of new material to students. They have found it necessary to teach algebra as the rote manipulation of skills rather than having sufficient time to develop meaning for algebraic concepts (pp.195-208).

Davis divides learning algebra into two separate categories. One through understanding algebra concepts, and the other through memorization of the needed moves. Davis is not alone in his views about the way that algebra is taught. Booth (1988) points out, “the purpose of viewing algebra as the expression of relationships was
to enable students to solve a wide variety of problems, however, when algebra was viewed as the manipulation of symbols, students did not see this purpose for algebra” (pp. 20-32).

In the construction of a tall building it is impossible to build anything that will last without first taking the time to lay a deep and solid foundation. Constructing mathematic success is no different than the construction of a building. A student needs to be supplied with the time necessary to fully understand the concepts before successful further learning can continue. The first method that Davis brings up of learning through repetition and memorization with no background makes it difficult for students to understand what they are learning and how this applies to them. This may be a partial reason for the finding reflected in the TIMSS Report. Students being taught with good solid mathematics fundamentals may be more successful.

The timing of when algebra should be taught is directly linked to which of the two teaching methods is being used. If the manipulation of symbols was being used, then algebra was primarily taught in high school as a ninth grade course when memorization techniques are stronger. On the other hand, “when attention was given to content, pacing, and exploring algebraic ideas, middle grade students were successful in algebra” because the time was allocated to build a solid conceptual foundation (Lodholz, 1990, pp. 24-33).

The National Council of Teachers of Mathematics (NCTM)

Research was conducted to identify components that were of major importance in the learning of algebra. The National Council of Teachers of Mathematics formed a commission of mathematics educators in 1989 to identify mathematical areas of
importance for the future in grades K-12. The document that came out of that commission, *Curriculum and Evaluation Standards for School Mathematics*, has become a manual across the nation for change in mathematics education. Algebra is one of the topics outlined in the standards for grades 5-8, and the NCTM recommends increased emphasis in the teaching of algebraic concepts. "In grades 5-8, the mathematics curriculum should include explorations of algebraic concepts and processes so that students can understand the concepts of variable, expression, and equation" (NCTM, 1989, p. 102).

It is essential to start the introduction of algebraic concepts in early grades according to the NCTM 1989, *Curriculum and Evaluation Standards for School Mathematics*, as "a bridge between the concrete elementary school curriculum and the more formal mathematics curriculum of the high school. One critical transition is that between arithmetic and algebra. It is thus essential that in grades 5-8, students explore algebraic concepts in an informal way to build a foundation for the subsequent formal study of algebra" (p. 103).

There is no limit set on how early some people think algebraic concepts can be introduced. The 1989 *Curriculum and Evaluation Standards for School Mathematics* suggests even as early as kindergarten these ideas have a place in the classroom.

Activities in grades 5-8 should build on students' K-4 experiences with patterns. They should continue to emphasize concrete situations that all students investigate ... and make generalizations [about algebraic thinking]. Expanding the amount of time that students have to make this transition to more abstract ways of thinking increases their chances of success. By integrating informal algebraic experiences
throughout the K-8 curriculum, students will develop confidence in using algebra to represent and solve problems (NCTM, 1989, p. 104).

These statements made by the NCTM show support for the view to begin algebraic investigation and thinking at a much earlier age. At this young age algebra should be taught for understanding of the concepts in algebra, not just the manipulation of symbols. "Mathematics is more than a sequence of isolated skills to be memorized". The National Council of Teachers of Mathematics Assessment Standards for School Mathematics (1995) calls for a "shift" away from just mastering isolated concepts and procedures and toward using concepts and procedures to solve problems (p.3).

As a result of the recommendations of the NCTM, patterns are now explored at the elementary grade levels, and pre-algebra is the current seventh grade curriculum. This is followed with algebra in eighth grade and high school, which now allows students ample time to be immersed in the concepts.

Issues and Trends Pertaining to the Teaching of Algebra

Cognitive development.

Jean Piaget, one of the foremost researchers in child development, classified learners into four stages of cognitive development: sensorimotor (birth to about two years old), preoperational (two to about seven years old), concrete operational (seven to about eleven years old), and formal operations (about eleven to adult). The last two stages are the development stages of typical middle school students (Juraschek, 1983, p. 58).

Helen Bee (1981) points out "the concrete operational child can reason inductively, going from his own experience to a general principle. But he has trouble
going the other way, from a general principle to some anticipated experience. He has a
hard time imagining things he has never experienced and has an equally hard time with
abstract concepts when they are not linked to specific objects. In formal operations, the
child becomes able to manipulate ideas as well as objects and can approach problems
systematically, deductive logic appears” (p. 225).

Juraschek (1983) points out that a “typical middle school class will most likely
contain a few students who are concrete operational and a few who are formal
operational, with the majority of the students in some transitional phase between the two
stages”. This range of development in the classroom can make the teaching of concepts
increasingly challenging (p. 59).

Many researchers indicated that students’ learning evolves through stages and that
the use of concrete manipulatives is important to bridge the gap between concrete and
abstract reasoning. Bohan (1971) indicated that “most students need pictures of the
object (semi-concrete) and diagram (semi-abstract) models to guide them from the
concrete to the abstract” (p. 246). Based on the experiences of teaching mathematics to
the learning different students for many years, I agree with Bohan’s research. I have
noticed that my students usually need to use concrete objects until they are comfortable
enough to progress to pictures of objects. From the pictures they gradually begin to draw
diagrams to represent the problems at hand. The finally step is the use of symbols to
represent the concept.
Manipulatives in mathematics

From my experience teaching mathematics to learning different students, and from experience as an in-class support teacher in the general education math classroom, I have found most students are concrete in their thinking. The use of manipulatives has been very successful in getting across concepts to confused students. After years of not understanding a mathematical process, the manipulatives are necessary to break through and change the cycle of incorrect procedures done for many years. Simply put, the manipulatives are needed to take the student back to the beginning and reteach the concept.

Manipulatives do not have to be anything special, in a matter of fact, I find using simple or familiar objects demonstrates to the students that they too can come up with objects when needed to solve a problem. The uses of these manipulatives, or objects, are used to illustrate facts, ideas, or processes and will promote both understanding and retention. Bley and Thornton (1981) even suggest using students as manipulatives by having them “physically dramatize the big idea of an operation”. This could be acting out what is meant by adding, subtracting, multiplying or dividing.

In the article, Manipulative Materials (1984), Marilyn Suydam noted in the researching of students, grades one through eight, the use of manipulative materials had a higher probability of producing greater mathematical achievement than do non-manipulative sequences. Suydam found that when learning is broken into three stages, from concrete objects to pictures to symbols, traditional symbolic treatments are probably at a disadvantage when used without first using concrete objects or pictorial models. The study found that generally pictorial models are superior to symbolic concepts, however
pictures are rarely superior to concrete experiences in younger children (p. 27). Thomas Post (1988) states, “one major problem in schools is the fact that many children are asked to abstract mathematical ideas before they have the opportunity to experience them in concrete form” (p. 11).

Patrick Thompson (1994) explains the evolution of the use of manipulatives for teaching mathematics. Thompson refers to a number of studies on the effectiveness of using concrete materials in the article, Concrete Materials and Teaching for Mathematical Understanding. Fennema (1972) argued for the use of manipulatives with beginning learners while maintaining that older learners would not necessarily benefit from them. However, Suydam and Higgins (1977) reported a pattern of beneficial results for all learners. Sowell stated that “students who used manipulatives in their mathematics classes usually outperform those who did not. This benefit holds across grade level, ability level, and topic, given that using a manipulative makes sense for the topic. Manipulative use also increases scores on retention and problem-solving test” (Thompson, 1994, p. 252). Thompson (1994) reported consistent success in the use of concrete materials to aid students’ understanding. He stated that “today we find common agreement that effective mathematics instruction, in the elementary grades, incorporates liberal use of concrete materials. The use of concrete materials seems to be assumed unquestioningly” (p.246).

I found this evolution of thinking on the use of manipulatives interesting because my schooling in mathematics, in the 1960’s, had very little if any hands on concrete instruction in mathematics and was mostly abstract which made learning difficult. Now as a special education mathematics teacher, I have learned to use manipulatives daily to
help the learning different students to grasp the mathematics concept being taught and feel this is a far superior teaching style.

Just using concrete materials alone is not enough to guarantee success. One must look at the total instructional environment to understand effective use of concrete materials. Baroody’s article, *Manipulatives Don’t Come With Guarantees* (1989), points out that:

Simply using manipulatives does not guarantee meaningful learning. Like any tool, manipulatives must be used judiciously and carefully for good results. If used inappropriately or without skill, they may not get the job done. Thoughtful use of manipulatives entails asking questions as these: Can pupils use this manipulative in such a way that it connects with their existing knowledge and, is meaningful to them? Is the manipulative used in such a way that it requires reflection or thought on the part of students?” (p. 250).

**Algebra and manipulatives.**

Algebra is referred to as the ‘gateway’ course to higher education. The focus now is to provide students with this ‘key’ for the gate to unlock future success. Adler (1984) stated, “Elementary algebra crosses the chasm between the arithmetic of everyday life and the abstract symbolism of mathematics proper” (p. 79). Former methods of teaching algebra were suited strictly for abstract learners leaving out the younger more concrete learner. Herbet (1985) stated that “any subject can be taught effectively in some intellectually honest form to any child at any stage of development. Hence, many younger students can learn some of the basic abstract concepts of algebra through games, and using manipulatives. Manipulatives motivate students: manipulatives stimulate
students to think mathematically, and manipulatives informally introduce "big ideas" in mathematics" (p. 4).

Robert Reys (1971) did a study on manipulatives and teaching mathematics. He suggests the following criteria in selecting meaningful manipulative materials for classroom use:

1. The manipulative should provide a true embodiment of the mathematical concept or idea being explored.
2. The materials should clearly represent the mathematical concept.
3. The materials should be motivating.
4. The materials should be multipurpose if possible (appropriate for several grade levels and different levels of concept formation).
5. The materials should provide a basis for abstraction.
6. The materials should provide for individual manipulation (p. 553).

In an article about rethinking concrete manipulatives, the following are suggestions offered by Evelyn Sowell to assist teachers implementing manipulatives in the classroom once a proper one is chosen.

1. Increase the students' use of manipulatives.
2. Recognize that students may differ in their need for manipulatives.
3. Encourage students to use manipulatives to solve a variety of problems and then to reflect on and justify their solutions.

Manipulatives are clearly helpful for developing algebraic ideas in students, but are not to be kept static as the only form of instruction throughout the students' progression. Marilyn Suydam (1984) offers one caution: "Not all children need to use
manipulatives for the same amount of time. Prolonged use may keep some children using procedures too simple and inefficient for them. Concern for individual needs must govern the use of manipulative materials” p. 27).

Various approaches using manipulatives to teach algebra.

There are many manipulative approaches to teaching algebra and the understanding of variables. One of the many programs and techniques researched is by Vollrath and Austin (1989). They suggest the use of a pan balance, small objects of equal weight, and small opaque containers to solve equations and introduce the students to the concept of variables (Vollrath & Austin, 1989, p. 608).

Another approach is by Berman and Friederwitzer (1989). They indicated that concrete models were needed for students to understand mathematical concepts. They used an activity involving envelopes marked with letters to signify an unknown quantity. Instead of a balance, students were taught to think of two islands in which the same thing was to be added or taken away from both. Students used manipulatives and diagrams to bridge the gap between arithmetic and algebra, and the results of the study were that elementary and middle school students could understand algebraic concepts when concrete models were used (p. 21).

Peck and Jencks developed a teaching approach that helps students make explicit links between arithmetic and nonnumerical notation of algebra. “To help students construct meaningful mathematics internally for themselves, physical materials were necessary. Students used graph paper, and strips of masking tape to model work. The teacher was a question asker, never an explainer nor judge of right or wrong.” They continue to say that “far too often, students are expected to find meaning in mathematics
not through their own reasoning but by reacting to the reasoning of teachers, and textbooks” (Kieran, 1991, p. 224).

Regardless of which study or approach is used, the one common thread between them all is use of tangible or concrete manipulatives in teaching. Lawson (1990) referred to the concrete approach towards algebra as, “new wave math”. He felt strongly that algebra needed to be taught with concrete materials before high school (p. 6).

The Hands-On Equation System.

The Hands-On Equations Learning System was another approach that provided concrete hands-on experiences to solve algebraic equations for students as young as third grade. Hands-On Equations is a learning system which incorporates physically representing equations by using a desktop set of manipulatives consisting of a laminated picture of a balance, blue and white pawns, and red and green number cubes. Using this set of game pieces, the student physically solves the equations by using legal moves. Students verbalize as well as pictorially represent what they are doing. (This will be examined fully in chapter 3.) Students learn intuitively that like terms can be combined without any discussion of this difficult abstract algebraic concept. The word variable is not used, but the blue pawn is named “X” (Borenson, 1994, p. 3).

Borenson (1994) indicated that “The Hands-On Equations Learning System is an algebraic learning environment which makes possible the Piagetian learning of algebraic concepts” (p. 4). “Piagetian learning refers to learning without being taught, as discussed by Seymour Papert in his book Mindstorm (1980, p. 7). Some of our most powerful and lasting learning, such as learning to speak, are carried out via Piagetian learning” (Borenson, 1994, p. 10). Borenson further stated that “early success with algebraic
concepts provided students with a tremendous sense of mathematical power and self-confidence, bolsters students’ mathematical interest, and provides students with an intuitive and concrete foundation for later algebraic work” (p. 2).

Borenson (1998) claimed “students are impressed with their ability to solve algebraic linear equations in a game-like manner. The “legal moves” provide students with a sound, intuitive understanding of fundamental algebraic properties without realizing they are learning them. The early acquisition of these concepts maybe an essential step in helping to raise the level of mathematics education in the United States. Students will experience success and enjoyment in solving what looks like sophisticated mathematics. This serves to enhance self-esteem and interest in mathematics. The Hands-On Equation System program is for students of average ability, gifted students, and for the student with learning disabilities. The visual and kinesthetic aspects of this program make it especially ideal for use with learning disabled students at any time from the fifth grade on” (p. 4).

Many researchers have indicated success in implementing an alternative instructional approach to aid students’ learning of algebraic concepts. In my opinion, based on the many years of teaching pull-out resource room mathematics, the use of concrete manipulative systems like this, has always been important for teaching concepts to the classified learning different student. I researched this Hands-On Equation learning system and attended an intense training workshop. I feel this system has merit and the potential to reach the classified students I teach in a new and innovative way.
Educational Factors for Special Education Students

History of the law of education for special needs students.

There have been many changes to the laws concerning the education of special education students. I feel these changes represent a fine-tuning of services for these special children. Knowing the history is beneficial for understanding where the laws began, and where they are heading. A teacher with this knowledge can hopefully learn from past standards, and present standards, to be a versatile instructor using various techniques. Sometimes keeping up with the changes can be burdensome, but ultimately these changes will bring a greater quality of education and life to these special students.

The history of educational law for special needs students began as recently as 1973, when legislation was created for individuals with disabilities, to provided them with education, employment, housing, and other rights that they previously were denied because of their disabilities. This landmark piece of legislation was enacted in 1975 and was known as the Education of the Handicapped Act, P.L. (Public Law) 94-142, which had significantly improved the opportunities for individuals with disabilities.

P.L. 94-142 was amended in 1986 to become known as P.L. 99-457, which ensured that all children with disabilities would have a free and appropriate public education available to them that would meet their unique needs.

In 1990 P.L. 99-457 was amended and the name was changed to the Individuals with Disabilities Education Act (IDEA), P.L. 101-476. According to Turnbull (1990), IDEA included the following primary characteristics:

- Zero Reject/Free Appropriate Public Education- No child with disabilities can be excluded from education. Mandatory legislation provides that all children
with disabilities are provided a free appropriate public education. Before IDEA, school officials who felt they were not equipped to address the special needs of particular students would not accept them into their school.

- Child Find- States are required to identify and track the number of students with disabilities and to plan for their educational needs.
- Age- IDEA provides for special programs and services for all students with disabilities between the ages of 3 and 21.
- Nondiscriminatory Evaluation- An evaluation that does not discriminate on the basis of language, culture, and students’ background.
- Individualized Education Program (IEP)- A plan developed to meet the special earning needs of each student with disabilities must be written, implemented, and reviewed.
- Least Restrictive Environment- IDEA defines the educational settings in which students are placed. The least restrictive environment is the setting most like that of nondisabled students, which also meets each child’s educational needs. These service options include self-contained classrooms, resource rooms, homebound, and general education programs.
- Due Process- This ensures that everyone with a stake in the student’s educational success has a voice. This includes written notification, testing for special education, parental consent, and guidelines for record keeping and appeals.
- Confidentiality of Records
- Advocacy- Advocates are assigned for individuals with disabilities who lack known parents or guardians.

- Noncompliance- IDEA requires that states mandate consequences for failure to comply with the law.

- Parent Participation- Parents' participation and shared decision making must be included in all aspects of identification and evaluation (p. 5).

Turnbull (1990) summarized the difference from the original legislation P.L. 94-142 and the Individuals with Disabilities Education Act (IDEA) of 1990, P.L. 101-476 (above), in that it:

- Establishes "people first" language for referring to people with disabilities.

- Extends special education services to include social work and rehabilitation services.

- Extends provisions for due process and confidentiality for students and parents.


- Requires states to provide bilingual education programs for students with disabilities.

- Requires states to educate students with disabilities for transition to employment, and to provide transition services (p. 6).

The latest change to the law was in 1997 when there was a reauthorization of the Individuals with Disabilities Education Act. The name IDEA remained the same with a change in Public Law number to, P.L. 105-17. Six main changes were:
• Requires that all students with disabilities must continue to receive services, even if they have been expelled from school,
• Allows states to extend their use of the developmental delay category for students through age 9,
• Requires schools to assume greater responsibility for ensuring that students with disabilities have access to the general education curriculum,
• Allows special education staff who is working in the mainstream to assist general education students when needed,
• Requires a general education teacher to be a member of the IEP team,
• Requires students with disabilities to take part in state and district wide assessments (Turnbull, 1990, p. 6).

The state of New Jersey follows the federal law mandated, and has established policy and procedures for statewide implementation for special education. It is called the New Jersey Administrative Code: Title 6A, Chapter 14, or NJAC 6A: 14.

Statewide standards for all areas of education are set forth in a document called the Core Curriculum Content Standards (CCCS). When the special education student’s IEP is created, there must be a statement of measurable annual goals that are related to the CCCS. As a special education teacher in New Jersey, there are many rules, regulations, and law that must be considered in the education of the special needs student. Knowing the law and CCCS is necessary to provide the student with the best possible education, as well as a framework and guidelines in which to implement any new teaching techniques.
Least restrictive environment.

The writers of the P.L. 94-142 recognize that successful handicapped adults have learned to function comfortably in the larger society of the non-handicapped world. The intent of the law was to help prepare handicapped students for that integration through experiences in school with non-handicapped peers. Public Law 94-142, which is referred to as the least restrictive environment, was designed to assure that this invaluable integrated experience is considered for all special needs students. IDEA defines the least restrictive environment as the setting most like that of nondisabled students, which also meets each child’s educational needs (Turnbull, 1990). In deciding the best setting for a special needs student, the general education classroom is always considered first. The next setting considered would be the general education classroom with in-class support provided by a special education teacher. The next level of placement would be a part-time placement in a special education resource room setting, for specific subjects, along with placement in the general education classroom with or without in-class support of a special education teacher. Level four placement would be a full time self-contained special education classroom within the general education school. This could include involvement with general education classes such as physical education, lunch, and music. Level five would be a special school, and level six would be a residential school, treatment center, or homebound instruction.

There is a nationwide trend towards reducing self-contained classrooms, and even pullout resource room settings. Unfortunately, for some students with serious learning disabilities, placement into the general education setting may become a complex, failure-
producing situation. There is no general rule, but in many cases, alternative or a combination settings, has been determined by some to be the best solution (Lerner, 1989).

The controversy of the least restrictive environment is one that continues to spark debate. According to Anderson (1998), current research on “compensatory and remedial education programs show that their goal of bringing academically deficient students back into the academic mainstream is not being accomplished” (p. 6). Some reasons include: teaching to the current level of academic functioning rather than to the levels they will actually need, lower levels of expectation and demand placed on the students, poor coordination of general and remedial programs, and a lack of effective educational leadership.

I teach in the Little Falls School System, which works very hard to maintain students with special learning needs in the general education classroom with support.

The math group that I will be doing this teacher action research with contains a majority of students recently moved from a self-contained classroom, and placed in a general education classroom with in-class support. These students also have pullout resource room for the subjects of mathematics, literature, and language arts.

Classifications and the learning different student.

Only in the last 27 years have learning disabilities been recognized in our schools. The issue of how to define learning disabilities has received considerable attention since1963, when an organizational meeting of the Learning Disabilities Association began the writing of legislation that is still used today.

The operational guidelines for the definition of specific learning disabilities are specified in the rules and regulations of IDEA. Learning disabilities represent a
heterogeneous group of students who despite adequate cognitive functioning have
difficulty learning. These students display a range of characteristics from low
performance in one or more academic areas, to unexpected low performance considering
their over-all ability, to ineffective or inefficient information processing. A
multidisciplinary team, often called the Child Study Team, may determine that a child
has a specific learning disability if that child shows any of these specified characteristics
(Vaughn, 2000, p. 158).

According to the Sixteenth Annual Report to Congress on the Implementation of
the Individuals with Disabilities Education Act (United States Department of Education,
1995), 10 percent of school-aged children were identified as disabled, and over 5 percent
were identified as having learning disabilities. In 1993-1994 this represented 2,444,020
students with learning disabilities that were identified and being serviced in the public
school system. Today there are even more classified students in the public school
system.

Classifying students is an area of controversy. Many people question the end
product of classifications, or the label that is attached to the child. As a special education
teacher, I feel that there are several problems involved with labeling children by their
disabilities. Often the label becomes the person rather than one facet of the person. The
label also affects the child’s self-image in a negative way and labels can increase social
discrimination. But, to ensure every child entitled to special education receives it, and
that there is the proper allocation of funds for special education, I realize it has become a
necessary evil to classify or label the student.
Teaching mathematics to the learning disabled student.

According to Vaughn (2000), it is not surprising that students with learning disabilities typically score below their same-aged peers on measures of math achievement. Some of their difficulties in mathematics relate to understanding the problem, and other instances they lack the computation skills. Interestingly, not all of their difficulties in mathematics relate to their knowledge of math, some relate to motivation. There is increasing evidence that these students have a poor working memory to retrieve information that would aid in solving problems. Also it is unknown whether students have difficulty learning the information and thus never storing it in long term memory, or whether students have difficulty accessing the information (p. 433). There are several variables to what is impeding these learning challenged students, but what is definitely known is that it is a teachers challenge to get past these blocks in the road.

In Teaching Mathematics to the Learning Disabled, Bly and Thorton (1981), stated that learning disabled children typically have average or above average intelligence. Teachers may mistakenly think they are lazy, not paying attention, or just not trying. These students are frequently misunderstood, and the learning different students may have mixed feelings about themselves. They do not look different, and work just as hard as others do, but they still cannot achieve in mathematics. Enrolled in a resource room, they often feel marked. Even when placed in mainstream situations they feel left out. Understanding a child’s disability sometimes helps teachers plan instruction to minimize frustrations and social pressure due to the handicap. The major disabilities that hinders success in mathematics have been divided into several general topics: visual
and auditory perception problems, discrimination, reversal, spatial and temporal disabilities, sequential difficulty, expressive and receptive language, abstract reasoning, motor, memory, and behavioral deficits (Bley and Thorton, p. 3).

**Self-esteem and the learning different student.**

Trying to foster a positive self-esteem in a learning different student is a very real problem. Based on my many years of teaching, I have found the student who has failed to learn for one reason or another tends to have low expectations of success, does not persist on tasks, and develops low self-esteem. These attitudes reduce motivation and create negative feeling about schoolwork. Walz and Bleuer (1992) defined self-esteem as:

The innate sense of self-worth that presumably is our birthright. More specifically, self-esteem is (a) confidence in our ability to think, confidence in our ability to cope with the challenges of life; and (b) confidence in our right to be happy, the feeling of being worthy, deserving, entitled to assert our needs and wants and to enjoy the fruits of our efforts. ... The higher our self-esteem, the more ambitious we tend to be, not necessarily in a career or financial sense, but in terms of what we hope to experience in life—emotionally, creatively, spiritually (p. 17).

Linda Campbell (1997) declared, “A school is responsible for helping students discover and develop their talents or strengths. In doing this, the school not only awakens children’s joy in learning but also fuels the persistence and effort necessary for mastering skills and information” (pp. 14-19). I completely am in agreement with Campbell and feel that it is a core goal of my daily teaching. I focus my
efforts on building my student’s self-esteem by helping them understand that people learn differently. By using books and magazines, the students learn about popular sports figures, actors, singers, CEOs, and intellectuals, like Einstein, who all struggled with difficulties. These students need to believe they can learn, and that they can make mistakes in safety. I have found that this confidence level is necessary for mathematics learning to take place. Children need to realize their learning difficulty affects only a small part of one intelligence area, leaving unimpaired vast regions of unlimited learning potential. These students need to be focused on their strengths. I feel a hands-on approach to algebra will be fruitful in not only teaching fundamental concepts but also to allow a platform for students to build confidence in their ability to solve any equation.

Summary of the Literature

It is unquestioned that the study of mathematics is important to ones future problem solving ability in life and potentially a career. Algebra is so central to mathematics that if students do not learn it, and learn it properly, their future options are severely limited (Willis, 1993).

Research has illustrated many difficulties involved in the learning of algebra (Booth, 1988). A major theme throughout indicates the need for algebraic concepts to be taught at an earlier age than the traditional ninth grade algebra course (NCTM, 1995, Usiskin, 1988). It was also clear that the teaching of algebraic concepts needs to incorporate meaningful experiences using hands on activities (Thompson, 1994). Research further shows that the concept of variables is one of the most important algebraic concepts to be taught and can be successfully taught in the middle grades
(NCTM, 1989). There are numerous instructional approaches that can be employed to aid in teaching the understanding of variables (Vollrath and Austin, 1989, Borenson, 1994).

The TIMSS Report (2000) indicated that most students in the middle grades are not functioning at a formal level of reasoning. The use of concrete manipulatives is necessary to extend the level of reasoning from concrete to formal and improve upon these negative findings (Piaget in Juraschek, 1983).

Finally, there are federal and state laws that protect and provide a free and appropriate public education for handicapped children. These laws have gone through several amendments to provide the best possible education for the special needs student, such as the addition of the least restrictive environment (IDEA, P.L. 101-476). One negative result that occurs over labeling classified students is the lack of self-esteem these students may exhibit. Research has also established expected mathematics difficulty for the student with learning differences (Bley and Thorton, 1981).

Overview for Chapter Three

Chapter three will contain various aspects of the plan to be used in this teacher action research. It will include definitions, demographics, who the participants will be, data collection tools, and the schedule of instruction. Chapter three will also state the limitations, and the kinds of validity to be used.
Chapter 3

Research Procedure

Introduction

Teaching pre-algebra is now one of the essential elements prescribed in the seventh grade mathematics curriculum. Many students this age are still primarily concrete learners. To provide a concrete approach this research study will examine a commercial manipulative program called Hands-On Equations by Dr. Henry Borenson. This is a novel, game-like teaching method. The purpose of this study is to look at the effects a manipulative hands-on approach to understanding pre-algebra has on a small group of seventh grade classified students with learning differences.

This chapter will contain various aspects of the study including definitions, demographics, participants, procedure of the study, schedule of instruction, and description of the manipulatives used in this research. The chapter will conclude with the data collection tools used for validity, and the limitations of this study.

Definitions for Study

(1) Manipulatives: Manipulatives will be defined in this study as objects which represent mathematical ideas that can be abstracted through physical involvement with the objects. The student is able to feel, touch, handle, and move the objects. They may be real objects that have social application in everyday affairs, or they may be objects that are used to represent an idea (Young, p. 12).

(2) Hands-On Equations: The commercial instructional program, and materials, used in the study are called Hands-On Equations designed and developed by Borenson. The
concepts taught are based on algebraic equations. The use of pawns, numbered
cubes and a balance beam are manipulatives that are governed by rules known as
"legal moves" (Borenson, 1994).

(3) Legal Move: Game like moves used to physically manipulate both sides of the
equation, while maintaining balance, to simplify the equation until the solution is
obtained (Borenson, 1994).

(4) Concrete: In this study concrete implies dealing with a tangible object relating to a
specific task, characterized by or belonging to immediate experience of actual things
or events (Webster, 1994, p. 239).

(5) Pictorial: Pertaining to diagrams or drawings that represent concrete objects or
actions.

(6) Abstract: Something disassociated from any specific instance or object but that
represents a family of ideas or events (Webster, 1994, p. 5).

(7) Kinesthetic/Tactile: Relating to the sense of touch (Webster, 1994, p. 1200).

(8) Multisensory: Involving many senses such as visual, auditory, and tactile.

(9) Learning Different Student: A child who, despite average intelligence, does not
respond to normal instruction with achievement reflective of his intelligence. The
term learning different and learning disabled are frequently used interchangeably
(IDEA PL 101-476).

(10) Equation: A mathematical sentence that contains an equal sign.

(11) Equal: symbol =, meaning one side of an equation is the 'same as' or 'even to' the
other side. Both sides are balanced.
(12) Inequality: symbol \(\neq\), meaning one side of an equation is not equal to the other side of the equation.

(13) Constant: a number that has a fixed value assumed not to change, in a given mathematical discussion (Webster, 1994, p. 247).

(14) Coefficient: any of the factors of a product considered in relation to a specific factor or a number used to multiply (Webster, 1994, p. 222).

(15) Variable: a quantity that may assume any one of a set of values, a value represented by a symbol like \(X\) (Webster, 1994, p. 1306).

**Demographics**

This study will be conducted at Little Falls Middle School, which consist of grades five through eight. The community is small and friendly with a population of about 11,200 people within an area of 2.5 square miles. The socioeconomic level is middle class and predominantly white, 88.4%, Hispanic, 5.3%, and Asian, 4.2%. The English speaking school population is 98%, with 2% classified as limited English proficient (U.S. Census Bureau, 2000).

This teacher action research plan will be conducted during the fall of the 2002. It will take place first period every day, Monday through Friday. First period begins at 8:39 a.m., and ends at 9:24 a.m. The research results will be based on data collected over a time period of approximately six weeks. Data collection tools will be used during the first unit of study, Level I, of the commercial product *Hands-On Equations*. Student success with the ground level skills established in Level I will be critical to the continuation of this program.
Participants

This study is based on five classified students in the seventh grade. All five students are in the general education system with in-class support for specific subjects. Due to their learning differences, they are pulled out from the mainstream class for replacement instruction in mathematics, language arts, and literature. This instruction will be provided in a resource room setting. The students' age range from twelve years old to thirteen years five months old. There are two females and three males in this group. Letters A, B, C, D and E will replace names of the students. The following chart (see Table 1) will give specific information about each of the participants.
<table>
<thead>
<tr>
<th>Student and Present Age</th>
<th>Classification &amp; Additional Services</th>
<th>2002 Local Composite Math % Terra Nova</th>
<th>Math grades from 6th grade</th>
<th>Days Absent In 6th grade</th>
<th>Additional Educational Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 13 yrs. 5 mon.</td>
<td>Specific Learning Disability (SLD)</td>
<td>7%</td>
<td>B+, E+ A-, B</td>
<td>8.5</td>
<td>1st year not in self-contained Language Learning Disability Classroom (LLD)</td>
</tr>
<tr>
<td></td>
<td>Speech &amp; Lang.</td>
<td></td>
<td></td>
<td></td>
<td>Extremely anxious, difficulty making transition to mainstream</td>
</tr>
<tr>
<td></td>
<td>Anger Management</td>
<td></td>
<td></td>
<td></td>
<td>Frustration levels can result in aggression</td>
</tr>
<tr>
<td>B 12 yrs. 7 mon.</td>
<td>Communication Impaired (CI)</td>
<td>3%</td>
<td>B, B, C, B</td>
<td>10.5</td>
<td>Came out of self-contained LLD classroom 6 months ago, good transition to mainstream</td>
</tr>
<tr>
<td></td>
<td>Speech &amp; Lang.</td>
<td></td>
<td></td>
<td></td>
<td>Non-verbal perceptual skills poor including spatial relations visual signs, scanning, numerical operations 3rd grade math reasoning 6th grade</td>
</tr>
<tr>
<td>C 12 yrs. 5 mon.</td>
<td>SLD</td>
<td>27%</td>
<td>B, B-, C+, C-</td>
<td>1</td>
<td>Came out of self-contained LLD classroom 1 yr. ago</td>
</tr>
<tr>
<td></td>
<td>Communication Impaired</td>
<td></td>
<td></td>
<td></td>
<td>Difficulty making transition and meeting work expectation</td>
</tr>
<tr>
<td></td>
<td>Obsessive Compulsive Disorder (OCD)</td>
<td></td>
<td></td>
<td></td>
<td>Anxious, emotional instability, fragile, depression</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Attention Deficit Disorder (ADD) no medication</td>
</tr>
<tr>
<td>D 12 yrs. 5 mon.</td>
<td>Other Health Impaired (OHI)</td>
<td>39%</td>
<td>B-, B-, C, B-</td>
<td>18</td>
<td>Came out of self-contained LLD classroom 6 months ago</td>
</tr>
<tr>
<td></td>
<td>Obsessive Compulsive Disorder (OCD)</td>
<td></td>
<td></td>
<td></td>
<td>Good transition, puts forth maximum effort</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>ADD with anxious qualities Takes RX</td>
</tr>
<tr>
<td>E 12 yrs.</td>
<td>SLD</td>
<td>11%</td>
<td>C, B-, B, B-</td>
<td>4</td>
<td>Never in self-contained Can be oppositional and resistant to ordered procedures, rules, steps</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Social issues with peers</td>
</tr>
</tbody>
</table>

Table 1. Specific Educational Information about Participants in the Study
Procedure of the Study

Five seventh grade classified students, with various learning differences, will be the focus of this teacher action research project. Before starting the *Hands-On Equations Program*, the students will fill out a survey to determine if there is a math anxiety level and what their general attitude is about mathematics (Appendix A). For the teacher to get a more personal understanding of how each student feels about mathematics, they will be individually interviewed using five questions (Appendix B).

To test the amount of previous knowledge related to algebraic equations and variables, and to assess the entry level of each student, a pretest of ten items will be administered (Appendix C). After the appropriate level of instruction is determined, a series of seven lessons from Level I of the *Hands-On Equation System* will be presented.

Each student will have an individual desktop set of manipulatives to physically maneuver, guided by the *Hands-On Equations System*. The resource room mathematics teacher will follow the seven written lessons contained in the Level I manual (Appendix D). Each lesson will be introduced and demonstrated by the teacher, or by use of the videotape, which is included with the program. Every lesson will be followed by a corresponding worksheet specifically designed for that lesson (sample, Appendix E). Problems on the worksheet will be solved independently by the students using the *Hands-On Equations’* manipulatives. The teacher, taking on the role of a coach, will circulate and observe that the students are using the proper techniques introduced by the present lesson. The coach will select a student that has done the work on a problem correctly to demonstrate (with the oversized balance and pieces), and explain how they physically
maneuver the pieces to find a solution for the blue pawn, or ‘X’. This demonstration will also include checking by setting up the original equation and proving each side of the scale is equal to the other side, or balanced. Then students will complete the remainder of the worksheet independently while the coach observes.

To assist students with retention of lessons, the videotape demonstrating each lesson may be used as review. This videotape is an optional product by *Hands-On Equations*. Copeland (1984) points out that teachers must find ways of teaching other than “show and tell” in order for children to remember mathematical ideas (p. 356). Learning different students often need a multisensory approach. Some are visual learners, others are auditory, while others are kinesthetic/tactile oriented learners. To see, hear and physically participate in solving equations will help the learning different students with processing difficulties, as well as promoting better memory of the concepts and skills learned. I feel presenting lessons in two formats will help students grasp the goal of each lesson.

When all the students complete the worksheet for the present lesson, they will take turns demonstrating each problem and explaining the process used to solve the equation. Additionally, students will be asked to journal write their thoughts about the program as they complete each lesson and new concept. Journal prompts and entries can provide valuable information and is a way to collect data from the students. This data will be used in a variety of ways.

1) Journal entries will provide a window into the students’ inner thoughts. Often the classified students lack the confidence to speak out in the presence of other classmates.
2) An important result of using journals is the development of the skill of reflection.

3) Journals will allow students to express their feelings as to whether they like the program.

4) Using a concept specific question could be used to evaluate whether the concept has been mastered.

5) Asking students to explain their thought process and method of solution is a powerful tool that will allow students to develop the skills necessary for higher order thinking. This type of prompt will help prepare students for the open-ended Grade Eight Proficiency Assessment (GEPA), which they will be taking next year.

Word problems will be introduced following lesson six and again after lesson seven. These will be demonstrated using algebraic equations, and the techniques learned through the *Hands-On Equation System*. The word problems will reveal a practical use of the new system they have been learning. I feel students need to make a connection between what they are learning and how the new techniques impact everyday activities. This is confirmed by Carnine, Dixon and Kameenui (1994) when they noted, "when learners integrate knowledge, they understand how it compares, contrasts, and fits with the rest of what they know. This improves their ability to apply it"(p. 1-3).

At the conclusion of the seven lessons, a posttest (Appendix F) will be administered to assess mastery of the concepts. The posttest will be in the exact same format as the pretest, containing ten items. The students will also be given the same survey (Appendix A), and be interviewed using the same questions that were used at the
beginning of the program (Appendix B), to identify any change in feelings about mathematics. There will be a group wrap-up discussion about the program and how they feel about pre-algebra. The group will decide if the program should be continued to Level II

**Schedule of instruction.**

The following chart (see Table 2) will outline the schedule of instruction by the day, objective for each day, and the procedure for the seven lessons to be taught in Level I of the *Hands-On Equation System.*

<table>
<thead>
<tr>
<th>Day 1</th>
<th>Pre-Program Student Information</th>
<th>Student Survey</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 2</td>
<td>Teacher Student Interview</td>
<td></td>
</tr>
<tr>
<td>Day 3</td>
<td>Pretest</td>
<td></td>
</tr>
<tr>
<td>Day 4</td>
<td>Lesson 1- Students become familiar with system</td>
<td>Teacher Demonstration Worksheet #1</td>
</tr>
<tr>
<td>Day 5</td>
<td>Lesson 1- Review</td>
<td>Video Complete Worksheet Student Demonstrations Journal Write</td>
</tr>
<tr>
<td>Day 6</td>
<td>Lesson 2- Students learn to set up equations from actual algebraic notation</td>
<td>Teacher Demonstration Worksheet #2</td>
</tr>
<tr>
<td>Day 7</td>
<td>Lesson 2- Review</td>
<td>Video Complete Worksheet Student Demonstrations Journal Write</td>
</tr>
<tr>
<td>Day 8</td>
<td>Lesson 3- Introduce ‘legal move’ for the variable</td>
<td>Teacher Demonstration Worksheet #3</td>
</tr>
<tr>
<td>Day 9</td>
<td>Lesson 3- Review</td>
<td>Video Complete Worksheet Student Demonstrations</td>
</tr>
<tr>
<td>Day</td>
<td>Lesson</td>
<td>Activity Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------</td>
<td>--------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Day 10</td>
<td>Lesson 3</td>
<td>Catch-up day to complete any unfinished work</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Journal Write</td>
</tr>
<tr>
<td>Day 11</td>
<td>Lesson 4-</td>
<td>Teacher Demonstration</td>
</tr>
<tr>
<td></td>
<td>Introduce ‘legal'</td>
<td>Worksheet #4</td>
</tr>
<tr>
<td></td>
<td>move’ for the</td>
<td></td>
</tr>
<tr>
<td></td>
<td>constant(numbers)</td>
<td></td>
</tr>
<tr>
<td>Day 12</td>
<td>Lesson 4-</td>
<td>Video</td>
</tr>
<tr>
<td></td>
<td>Review</td>
<td>Complete Worksheet</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Student Demonstrations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Journal Write</td>
</tr>
<tr>
<td>Day 13</td>
<td>Lesson 5-</td>
<td>Teacher Demonstrate</td>
</tr>
<tr>
<td></td>
<td>Introduce the</td>
<td>Worksheet #5</td>
</tr>
<tr>
<td></td>
<td>concept of ‘minus</td>
<td></td>
</tr>
<tr>
<td></td>
<td>X’</td>
<td></td>
</tr>
<tr>
<td>Day 14</td>
<td>Lesson 5-</td>
<td>Video</td>
</tr>
<tr>
<td></td>
<td>Review</td>
<td>Complete Worksheet</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Student Demonstrations</td>
</tr>
<tr>
<td>Day 15</td>
<td>Lesson 5</td>
<td>Journal Write</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Catch-up day for unfinished work</td>
</tr>
<tr>
<td>Day 16</td>
<td>Lesson 6-</td>
<td>Teacher Demonstration</td>
</tr>
<tr>
<td></td>
<td>Introduce the</td>
<td>Worksheet #6</td>
</tr>
<tr>
<td></td>
<td>use of</td>
<td></td>
</tr>
<tr>
<td></td>
<td>parenthesis ( )</td>
<td></td>
</tr>
<tr>
<td>Day 17</td>
<td>Lesson 6-</td>
<td>Video</td>
</tr>
<tr>
<td></td>
<td>Review</td>
<td>Complete Worksheet</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Student Demonstrations</td>
</tr>
<tr>
<td>Day 18</td>
<td>Lesson 6</td>
<td>Catch-up day for unfinished work</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Journal Write</td>
</tr>
<tr>
<td>Day 19</td>
<td>Word Problems</td>
<td>Use word problems to set up equations and solve using new techniques</td>
</tr>
<tr>
<td>Day 20</td>
<td>Review lessons 1-6</td>
<td>Use video to review lessons 1-6</td>
</tr>
<tr>
<td>Day 21</td>
<td>Lesson 7-</td>
<td>Teacher Demonstration will show how to draw balance, pictures of pawns, and pictures</td>
</tr>
<tr>
<td></td>
<td>Pictorial</td>
<td>of number cubes</td>
</tr>
<tr>
<td></td>
<td>solutions, going</td>
<td></td>
</tr>
<tr>
<td></td>
<td>from concrete to</td>
<td></td>
</tr>
<tr>
<td></td>
<td>pictorial</td>
<td></td>
</tr>
<tr>
<td>Day 22</td>
<td>Lesson 7-</td>
<td>Video</td>
</tr>
<tr>
<td></td>
<td>Review</td>
<td>Worksheet #7</td>
</tr>
<tr>
<td>Day 23</td>
<td>Lesson 7</td>
<td>Complete Worksheet</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Student Demonstrations</td>
</tr>
<tr>
<td>Day 24</td>
<td>Lesson 7</td>
<td>Student Demonstrations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Journal Write</td>
</tr>
</tbody>
</table>
Day 25 | Word Problems | Use word problems to set up equations and solve using hands-on or pictorial technique
---|---|---
Day 26 | Post Program Student Information | Posttest
Day 27 | | Student Survey
Day 28 | | Teacher Student Interview
Day 29 | Word Problems | Work some more word problems using either technique learned
Day 30 | Group Discussion | Group discussion about Level I Should the program be continued to Level II

Table 2. Daily Schedule of Instruction

Description of manipulatives.

From the Hands-On Equation Learning System, each student will have a desktop set of manipulatives consisting of a laminated picture of a balance scale, eight blue pawns (similar to chess game pieces), and four red number cubes. Two of the cubes are numbered 0 – 5 and two cubes are numbered 5 – 10. The instructor will use an oversized stationary balance model on which to set up an oversized set of manipulatives for class demonstration. This same balance and pieces will be used when students demonstrate solving equations for the group.

Each blue pawn represents a variable, designated as ‘X’. The number cubes represent the constant (not coefficients). The pictorial balance (see Table 3) represents
the idea of a balanced equation. The students will start with a balanced system, and the object is to determine the value of the 'X' while maintaining the balanced system. The representation for the problem $4x + 2 = 2x + 8$ would be:

Table 3. Pictorial Balance

Students will learn to simplify the equation by using the ‘legal move’ presented in lesson three (see Table 4). The students will learn to simplify by removing the same number of pawns from each side of the balance, thus keeping the equation in balance.

Table 4. ‘Legal Move’ with Pawns

Each lesson will build on the lesson before. The next ‘legal move’ found in lesson four (see Table 5), the students will learn will to remove the same number amount from the number cubes on each side of the balance. So that:
2x + 2 = 8  
\[\text{simplifies to}\]  
\[2x = 6\]

Table 5. 'Legal Move' with Number Values

From this representation, the students will determine the value of each pawn to be 3, thus \(x = 3\). The system will emphasize the importance of checking the value of 'x' against the original equation set-up to ensure all moves are legal and the correct value of 'x' has been determined. Students will set up the original equation and then check to see if it balances when each pawn is assigned the value that has been determined (see Table 6).

\[
\begin{array}{c|c}
3 + 3 + 3 + 3 + 2 & 3 + 3 + 8 \\
\end{array}
\]

When 'x' = 3, the solution is \(14 = 14\) and the system is in balance.

Table 6. Original Set Up with Check
Data Collection Tools

Data collection will be an important part of the overall development of this teacher action research. Throughout the six weeks needed to complete the research, data will be collected in a variety of ways to provide a more complete picture of the results to answer the research questions, and to ensure that the results are valid. The collection tools that I will be using are based on triangulation to give this qualitative data analysis credibility. On the topic of triangulation, Marshall and Rossman explain, “Triangulation is the act of bringing more than one source of data to bear on a single point. Designing a study in which multiple cases are used, multiple informants or more than one data gathering technique can greatly strengthen the study’s usefulness for other settings” (Glanz, 1998, p. 41).

For a more accurate view of assessing the Hands-On Equation program, I feel it is critical to incorporate multiple sources of data. The data to be collected will be based on triangulation consisting of student assessment scores, student voice, and teacher observations.

One part of triangulation will be student assessment scores, which consist of existing records, and assessments that will be obtained during this program. Existing records will aid the teacher in obtaining a baseline on the student’s strengths and weakness pertaining to mathematics. These records will include the March 2002 school-wide standardized test, grade six report card attendance record and scores in mathematics, and each student’s IEP for information specific to the student’s learning needs (see Table 1). To collect fresh assessment data based on the program being introduced a pretest (Appendix C) and posttest (Appendix F) will be administered. These will be the exact
same format using ten equations. The worksheets (Appendix E) that accompany each lesson will also give concrete evidence of concept understanding or lack they’re of.

The second part of the triangulation method is to give the student a voice. This will include a student survey (Appendix A), one-on-one teacher student interview (Appendix B), journal writing, and a group discussion about the program. The student survey will be done before the program is introduced and again after the seven lessons are completed. This will provide information as to changes in the students’ attitude about math class. Likewise, the one-on-one teacher student interview will be done before and after the program. This will allow the students to convey their thoughts in a comfortable verbal exchange. The responses can then be compared to identify changes from before and after the Hands-On Equation program. The journal entries will provide student voice on a regular base that will be after every lesson and after the posttest. As the final activity for Level I of the Hands-On Equation program, there will be a group discussion. This will decide if this math class will continue with Level II of the program.

To complete the triangulation method, the third data analysis tool will be teacher observation. This will be recorded in the teacher’s daily log. The teacher will record observations that are about the lesson of the day, what problems occurred, and thoughts on how to improve the lesson. The log will also include observations about individual students. This log will keep track of the students’ attitude, participation, and class work. The log will also record the teacher’s individual thoughts about the Hands-on Equation System.

This will be a qualitative research in that the results can be interpreted only in relation to this study. It cannot be generalized to others students, classrooms, schools, or
educational settings. The purpose of this study is to find out what will happen with my small group of seventh grade classified students, learning pre-algebra in a resource room setting, while using a commercial program called Hands-On Equations. This plan and research will have validity because I will be using triangulation data collection. I will be using outcome validity, as the completion of this project will not be a means to an end, but an ongoing learning experience. Process validity will be considered, as there may be times when lessons may need to be adjusted to fit the setting and flow of the day. Catalytic validity will be used to determine any positive or negative influence this program has on the students and teacher. My students, colleagues, family, and the teacher action collaborative group will support me by sharing ideas, and knowledge, which is the use of dialogic validity (Anderson, Herr & Nihlen, 1994).

**Limitations**

There will be some limitations involved with this research. One limitation will be the physical setting of the resource room. The Little Falls School System is presently under construction. As a result, the usual location for this resource room is not available. This class will be taught in a different location each day of the week consisting of the cafeteria and four available classrooms depending upon the specific day of the week. The impact of this situation will be observed and documented in chapter four.

There are often the limitations of unforeseeable interruptions such as fire drills, unscheduled presentations, and assembly programs. With classified students, there are times when a student has a personal situation that will affect the lesson scheduled for that
day. When teaching learning different students, allowances may need to be made for specific students on disconcerting days.

Limitations due to student absenteeism may present a problem if a new lesson is being demonstrated. The student will need to make up the missed concept because each lesson builds on the previous lessons.

The time frame of the study will only show what the students have learned at the end of seven lessons. This program has twenty-four lessons spread between three levels. The six weeks will provide time for Level I, but the students will not be able to experience using the complete system beyond Level I during this study. The length of this study may not provide the time needed for observing whether the students’ attitude towards learning mathematics have changed. This observation may not occur for several months.

Finally, this group of participants is not representative of any other class of students. Results are limited to this particular group of seventh grade classified resource room students.

Summary

This chapter has gone into great detail about the plan to be executed in order to answer the research questions. Terms used in this study have been defined for a more complete understanding of this thesis report. Information about the demographics, and participants has been reported. The procedure used for this research, as well as the daily schedule of lessons, and a description of the manipulatives used in this commercial program has been explained. The chapter ends with detailed information about the
validity in this qualitative research, including data collection using triangulation, and the limitations of this study.

**Overview of Chapter 4**

Chapter four will reveal the findings from carrying out the outlined procedures, daily lessons, and data collection described in chapter three. Will these students be able to physically set up and solve given equations? How else will students be affected by using the *Hands-On Equation System*? Will there be a change in attitude about mathematics by the end of the seven weeks? These research questions will be answered with the results found in chapter four.
Chapter 4

Results

Introduction

This chapter will present the data collected in order to answer the research question of the impact the *Hands-On Equation System* has on seventh grade classified resource room mathematics students. To fully satisfy this question, specific areas will be examined, such as the students’ ability to physically set up and solve given algebraic equations, changes in the students’ attitude about algebra, and other ways students have been affected by using this system.

The data collected is based on triangulation to give this qualitative teacher action research credibility. The areas of triangulation will include student assessment scores, student voice, and teacher observation.

At the beginning of the research, students were given a mathematics survey (Appendix A), one-on-one teacher student interview (Appendix B), and a pretest for basic skills assessment (Appendix C). This data gathering process was repeated again at the end of the six-week study. Students were also required to do journal writing about each lesson upon its completion. Finally, a teacher’s daily log was kept containing in-depth observations about each student, as well as the strengths and weaknesses of the *Hands-On Equations Program*:

The subjects for this research work were five seventh grade students. These students are all classified with various learning differences (see Table 1). The instructional setting was a resource room, and I taught all the lessons myself to eliminate any extra variables. No explanation was made to the students concerning this research
study beforehand to ensure the integrity of the data, but the superintendent, special education administer, building principal, and other teachers were familiarized with this research study.

Lesson by Lesson Results

Following the student mathematics survey, one-on-one teacher student interviews, and the pretest, the *Hands-On Equations Program* was introduced. This program has three levels containing twenty-six lessons. Level I was examined in this research and contains seven lessons. Each of the seven lessons has detailed lesson plans that can be found in the appendix of this thesis (Appendix D). The following will summarize how each of the seven lessons was presented, student journal writing, and teacher observation about each lesson.

**Lesson 1.**

The teacher, using the demonstration balance and manipulatives, introduced lesson one. The students were first asked to visualize an equation as an equal balance, like a seesaw, with the equal sign as the fulcrum. They also were introduced to the idea of using the pawn to represent ‘x’, or the variable. Then the teacher modeled how to set up the equations on the demonstration balance using the manipulatives, and the finally finding what ‘x’ equals. These concepts are essential not only to lesson one, but also for the continuation of the subsequent lessons. During this class, six equations were demonstrated with students orally participating in the check, which consisted of addition and times tables.
The video demonstrating lesson one was shown during the second class meeting. This was a shortened period so only ten minutes were available for this class.

For the third class, the students wanted to watch the video again. Following this review the students were given their own laminated balance and manipulatives. They began the lesson one worksheet as the teacher acted as a coach for those who needed it. After six equations were completed individually, the students demonstrated the steps and results before the group.

During the fourth class meeting, all the students completed worksheet one and demonstrated the remaining four problems on the sheet. A perfect score was achieved by all the students on this worksheet. Students were then asked to journal write what they thought about the *Hands-On Equation Program*. This class period ended by previewing lesson two because the students were excited to know what was coming next.

Table 7 will show the teacher’s observation of each student, as well as the students’ journal writing on how they felt about the *Hands-On Equations Program* after finishing worksheet one.
<table>
<thead>
<tr>
<th>Student</th>
<th>Teacher's Observation</th>
<th>Students' Journal Writing</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>needed some help with written steps and organization, neatness was encouraged, when student understood what was expected completed the worksheet quickly on own</td>
<td>It was kind of scary when I was going to be in a regular classroom. It's been fun doing math and it's kind of easy doing this math. It's fun working in this math class.</td>
</tr>
<tr>
<td>B</td>
<td>student needed encouragement to write down his guess and check, could use this to compare values and find the correct answer, student then moved quickly on own to complete the worksheet</td>
<td>I think the program will be fun. I already like it.</td>
</tr>
<tr>
<td>C</td>
<td>student completed the whole worksheet on own, no coaching needed</td>
<td>I think the <em>Hands-On Equations</em> is fun and easy to learn.</td>
</tr>
<tr>
<td>D</td>
<td>student needed encouragement to write down guess and check, could use this to compare values and find the correct answer, then student moved more rapidly in solving equations on the worksheet</td>
<td>I like this program for many reasons. It is a lot of fun first period. With this program I have never had more fun in math.</td>
</tr>
<tr>
<td>E</td>
<td>student chose not to use manipulatives, instead the student looked at the pictorials to solve equations, student had to be encouraged repeatedly to write down guess and check, student had difficulty with the pictorial and was strongly encouraged to use hands-on manipulatives</td>
<td>I don't know what I think of this. It is kind of confusing.</td>
</tr>
</tbody>
</table>

Table 7. Comparison of Teacher's Observations and Students' Journal Writing for Lesson 1
Lesson 2.

Lesson two was introduced by watching the demonstration video. This was the second time the students viewed this video. Lesson two was an extension of lesson one and provided reinforcement of the concepts that are needed for future lessons. The students then worked on the worksheet and completed the ten equations during this class period. All students again achieved a 100% score on this worksheet.

The second class of this lesson was used for students to demonstrate all the equations. Since there was time remaining, the teacher began talking about the ‘legal move’, which would be introduced in lesson three.

Table 8 will show the teacher’s observation of each student, as well as the students’ journal writing on how they feel about the lesson just completed.

<table>
<thead>
<tr>
<th>Student</th>
<th>Teacher’s Observation</th>
<th>Students’ Journal Writing</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>student needed to be reminded the 2X meant two pawns, not the number 2 and a pawn, student did an excellent job completing the worksheet.</td>
<td>This is easy for me. I can do it.</td>
</tr>
<tr>
<td>B</td>
<td>student did excellently completed on own</td>
<td>This is fun and easy.</td>
</tr>
<tr>
<td>B</td>
<td>student did excellently completed on own</td>
<td>This is too easy for me. I do like to use the pieces. It’s fun.</td>
</tr>
<tr>
<td>C</td>
<td>student needed to be reminded that 2X meant two pawns, not the number 2 and a pawn</td>
<td>I think I am doing good with this math. I had to guess a lot but I can do it.</td>
</tr>
<tr>
<td>D</td>
<td>student still resistant to using the hands-on manipulatives, having trouble with the concept of a balance, coach had to physically set up equations on the laminated balance to get student to trust the pieces, did last five equations on own successfully</td>
<td>It’s okay I guess. This is very different math. Sometimes it is confusing.</td>
</tr>
</tbody>
</table>

Table 8. Comparison of Teacher’s Observations and Students’ Journal Writing for Lesson 2
Lesson 3.

Lesson three was introduced by watching the demonstration video. This lesson teaches the game-like maneuver called 'legal moves'. This is a very important concept in algebra, which allows a student to remove an equal amount of pawns from each side of the balance so not to disturb the balance, to simplify the equation and making it easier to solve. After viewing the video, students were given the worksheet. Student A was absent for the first class of lesson three.

At the beginning of the second class, the video was shown again for the benefit of the one absent student, but the other students felt they needed to watch the video also. Then all the students continued the trend of completing the worksheet with 100% accuracy.

The third day on this lesson was used to journal write and to let students demonstrate the equations. Most students were looking forward to acting as the 'teacher' by demonstrating and explaining the equations. The last five minutes of the period were used for the teacher to demonstrate lesson four, which increased the 'legal moves' to include number values, or constants, to solve equations.

Table 9 will show the teacher's observations of each student, and the students' journal writing on how they felt about the 'legal move' used in this lesson.
<table>
<thead>
<tr>
<th>Student</th>
<th>Teacher’s Observation</th>
<th>Students’ Journal Writing</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>student was absent when this lesson was introduced, had student watch others model ‘legal moves’, observed student doing three problems and then student completed the entire worksheet in one day, not sure student really understands concept</td>
<td>It was kind of hard to understand. When you said that you take a pawn from each side and it was very difficult for me to understand.</td>
</tr>
<tr>
<td>B</td>
<td>instead of removing the same number of pawns from each side student wanted to leave the same number of pawns on each side, had to explain the equation is in balance and student must maintain the balance by removing the exact same thing from both sides, had student use both hands to reinforce this concept, student completed on own successfully</td>
<td>I think the ‘legal move’ is a way to get out of a bind. I used the ‘legal move’ a few times.</td>
</tr>
<tr>
<td>C</td>
<td>student did excellently on this and needed no coaching</td>
<td>I think the ‘legal move’ made the work easier to do and it is a cinch to do.</td>
</tr>
<tr>
<td>D</td>
<td>student felt unsure of the ‘legal move’ so the teacher observed problems 1 and 2 being solved, then student completed remaining equations on own</td>
<td>I think the ‘legal move’ makes doing algebra equations easier and faster. I really liked learning how to do it. Doing this program had made math fun and ‘legal moves’ made it easier to understand and made it faster to do.</td>
</tr>
<tr>
<td>E</td>
<td>student very resistant to setting up manipulatives and using hands to do ‘legal moves’, student still computing in head instead of writing down guess and check, strongly insisted student follow the steps of the program and use both hands when executing the ‘legal moves’, student seems unsure of ‘legal move’, often just stares at problem for 5 minutes, observed three problems being solved then student completed on own.</td>
<td>I thought the ‘legal move’ was okay. It’s confusing though because it’s a new way of doing math. Maybe I’ll get it sometime.</td>
</tr>
</tbody>
</table>

Table 9. Comparison of Teacher’s Observations and Students’ Journal Writing for Lesson 3
Lesson 4.

Lesson four was introduced by watching the demonstration video. This lesson expanded the ‘legal move’ to include removing equal number values from each side of the equation so as not to disturb the balance. Students requested watching the video a second time, as they felt unsure of what to do. The teacher then demonstrated four equations using the large balance.

The second meeting was after a gap of four days, so the video was watched again as a review. The teacher reviewed the concept of ‘legal moves’ using both pawns (the variables), and number value cubes (the constants), on the large demonstration balance. Students were then given the worksheet to do.

The third class was used to complete the worksheet and to have students demonstrate the equations. All students achieved a 100% score on the worksheet. They really enjoyed demonstrating the equations because there was more thinking and actual movement of the game-like pieces. After simplifying, using the ‘legal moves’, the answer was often sitting on the balance and no other mathematical calculations were needed.

Table 10 will show the teacher’s observation of each student as recorded in the daily log, and the students’ journal writing on what they thought about the ‘legal moves’ in lesson four.
<table>
<thead>
<tr>
<th>Student</th>
<th>Teacher’s Observation</th>
<th>Students’ Journal Writing</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>student seemed a little unsure at start but completed worksheet successfully on own in one period</td>
<td>I think it was kind of hard to understand from the start but in the end it was easy.</td>
</tr>
<tr>
<td>B</td>
<td>student started quickly and did an excellent job on own, completed worksheet in one period, student demonstrated confidence in solving equations</td>
<td>I think the ‘legal move’ is a good thing to use. It gets you out of binds very easy. That is what I think about the ‘legal move’.</td>
</tr>
<tr>
<td>C</td>
<td>student confident and couldn’t wait to get started on the worksheet, only needed about twenty minutes to complete, student did an excellent job on worksheet</td>
<td>I think that it makes it easier and more fun to do and it helps me learn.</td>
</tr>
<tr>
<td>D</td>
<td>student was preoccupied with locker problems during class time, student was able to complete seven out of ten equations during this period, completed last three the next class, good job</td>
<td>Now that I understand the ‘legal moves’ the equations are fun and really easy. I like coming to math class.</td>
</tr>
<tr>
<td>E</td>
<td>student asked for help which is very rare, student needed to be talked through the steps and encouraged to use both hands when removing pawns and number cubes, student is now cooperative and is actually smiling as the answers are found, student is building some confidence due to arriving at the correct answer, sometimes the check doesn’t work because student adds incorrectly during the check</td>
<td>This is becoming fun now that I think I am getting it.</td>
</tr>
</tbody>
</table>

Table 10. Comparison of Teacher’s Observations and Students’ Journal Writing for Lesson 4
Lesson 5.

Lesson five was introduced by watching the demonstration video, and the students wanted to review the video a second time. They seemed to be excited at how easy this lesson looked on the video. Lesson five is the first time subtraction is seen in the equations. The subtraction is only used with the pawns, or ‘x’. Two students completed the worksheet successfully during the first class.

The second class on this lesson was used for three students to complete their worksheets. Two of these students needed coaching on a few equations, and then completed the worksheet on their own. All students achieved a 100% score on this worksheet.

The third class was used for students to demonstrate all ten equations for this lesson. The students now are having fun. This is observed by the laughter, smiling, and ease of the students during demonstration of the equations. The students are enjoying functioning as the ‘teacher’ and saying to other students, “Let’s all count out loud together for the check”. When the demonstrations were completed, the group previewed the video for lesson six.

Table 11 will show teacher observation of each student as recorded in the daily log, and the students’ journal writing on their thoughts about the subtraction of ‘x’ used in this lesson.
<table>
<thead>
<tr>
<th>Student</th>
<th>Teacher's Observation</th>
<th>Students' Journal Writing</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>student was having difficulty with the -X, kept thinking it was an = sign, teacher observed and coached student through a few problems then student completed the worksheet successfully</td>
<td>It was hard but then I started to understand it.</td>
</tr>
<tr>
<td>B</td>
<td>student completed worksheet in first class, student is confident and successful with equations</td>
<td>I think the subtraction is just like the addition. The reason I think that is because you still have to add but they make it easier.</td>
</tr>
<tr>
<td>C</td>
<td>student can't wait to start, physically moves away from the group for better concentration, finished in first class and did an excellent job, very confident as shown by the smile when handing in worksheet</td>
<td>The taking away makes pre-algebra fun to learn and faster to find the answer.</td>
</tr>
<tr>
<td>D</td>
<td>student worked slowly but was confident in own work, student only asked one question that being could an answer be zero</td>
<td>I really think that the subtracting is fun. This is the most fun thing I have ever done in math.</td>
</tr>
<tr>
<td>E</td>
<td>student had some difficult at first with the -X, thought that you do nothing because it says -X, teacher worked closely with this student in setting up the equation on student's balance, after two problems student seemed amazed at getting the correct answer, student completed the worksheet on own successfully</td>
<td>What I thought about taking away X's was kind of confusing. It was hard at first, but now I think I'm getting the hang of it.</td>
</tr>
</tbody>
</table>

Table 11. Comparison of Teacher's Observations and Students' Journal Writing for Lesson 5
Lesson 6.

Lesson six was introduced by watching the demonstration video two times. Students really like to view the lessons more than once. They know they need to have more time on the concept to understand it better and through the repeat viewing they have the confidence needed before beginning the worksheet. This lesson introduced an important concept, the use of parenthesis. The students learned that if the number outside the parenthesis (coefficient) is a two, then whatever is inside the parenthesis is to be doubled. Two students completed the worksheet during the first class. The other three students only had two problems remaining for the next class.

The second class allowed time for three students to complete their worksheets. All students achieved a perfect score on this worksheet. Then students demonstrated equations on the large balance. The group showed confidence and enthusiasm about doing the equations. All students readily verbalized the ‘legal moves’ with confidence. The students had fun while they acted as the ‘teacher’ by demonstrating, explaining, and requesting class participation.

Table 12 will show the teacher’s observation of each student as recorded in the daily log, and the students’ journal writing on how they felt about this lesson with parenthesis and the Hands-On Equation Program.
<table>
<thead>
<tr>
<th>Student</th>
<th>Teacher's Observation</th>
<th>Students' Journal Writing</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>student completed worksheet with no questions, good work</td>
<td>It was easy and the program is fun. It was easy when we had to double it up.</td>
</tr>
<tr>
<td>B</td>
<td>student worked confidently and completed work sheet in first class</td>
<td>This was a fun lesson because it was new. This program is fun and I see what it is suppose to do.</td>
</tr>
<tr>
<td>C</td>
<td>student worked confidently and completed worksheet in first class</td>
<td>The parenthesis made it a little bit difficult. I have enjoyed coming to math and doing these problems.</td>
</tr>
<tr>
<td>D</td>
<td>student completed worksheet with no questions, good work</td>
<td>I think that in the beginning it was a little hard. Then it got easier as time went on. I really love this program. It is so much fun.</td>
</tr>
<tr>
<td>E</td>
<td>student completed worksheet with no questions, good work</td>
<td>I thought it was easy. I feel it is okay. I understand a little bit about this. When all of this is done I'll know it all. I am surprised about the way I get the answers.</td>
</tr>
</tbody>
</table>

Table 12. Comparison of Teacher's Observations and Students' Journal Writing for Lesson 6
Lesson 7.

Lesson seven was introduced by watching the video three times, at the students' request. This lesson is important because the students learn to move from the concrete form by using the manipulatives, to a pictorial form by drawing the balance and the pieces. Once the video was viewed, worksheet seven was given out with paper to draw the pictorial equations to solve. No manipulatives were given out at this point. At the end of this class almost all the students were on number three.

For the second class meeting, all students were given the worksheet to complete up to number four. At this point the teacher would check over the pictorial equations and make corrections and/or suggestions. The students were then given the manipulatives with a choice to solve the remaining six problems with the manipulatives and/or pictorial drawings. As a matter of observation, all students chose to use the manipulatives.

The third class was spent on demonstrating how they solved the equations. Students were vocal about wondering why they had to do the drawings. They felt the drawings took more time than the manipulatives. When the teacher put the question to the students, "Why do you think this program is teaching you the pictorials?" one student was able to recognize that the numbers might be bigger than the manipulatives they had in their bag. The other students all nodded to show their agreement to this response and they seemed satisfied with this explanation.

The students were not asked to journal write after this lesson because they initiated discussion about lesson seven on their own, and journal write would have reflected the group discussion rather than an individual reflection. Also individual
student reflection was going to be accounted for during the one-on-one teacher student interview.

The teacher’s observation log indicated that three students had difficulty with problem eight \((2X + 1 + X + 4 = X + 16 + X)\). The students knew their answer was not correct because \(2X\) could not equal 11. Then \(X\) would equal 5.5 and at this time only whole numbers were used. When students were asked to show how they set up the equation, it was realized that the final \(X\) was left off the initial set up. They were all able to solve the equation at this point. There was a 100% score for all students on this worksheet.

Test Results from the Pretest and Posttest

Data was collected on each individual student by means of a pretest and posttest. The tests consisted of ten items in the same format, but the equations were not identical. Students were not offered the manipulative kit for the pretest, as they did not know how to use it yet. They were given the manipulative kit for use in taking the posttest. An individual score of 8 out of 10 (80%) or better was considered mastery of the concepts at this level.

All students had some previous instruction with linear equations from prior years of mathematics. This consisted of equations of the type \(x + a = b\), where \(a\) and \(b\) are integers. Solving for the missing addend has been taught since the first grade. This equation form was represented by items #1 and #2 on both the pretest and posttest administered (see Appendixes C and F). All five students got #1 correct on the pretest, but only two students got #2 correct. During the pretest no equation was correctly solved.
for #3 - #10 as the equations became increasingly difficult. In many cases those harder equations were met with frustration and not even attempted. The results in the posttest were strikingly different. The results from the pretest and posttest are recorded in Table 13.

<table>
<thead>
<tr>
<th>Student</th>
<th>Pretest Score</th>
<th>Posttest Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>80</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>90</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>E</td>
<td>20</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 13. Results of the Pretest and Posttest

Class's mean score for pretest and posttest.

The mean score for the pretest and posttest was obtained by totaling all the scores from the particular test, and dividing the sum by the total number of students. The quotient is the class's mean score. This is recorded in Table 14 below.

<table>
<thead>
<tr>
<th>Mean Score of Pretest</th>
<th>Means Score of Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 14. Mean Scores for Pretest and Posttest

Percent of students at mastery levels.

The percent of students demonstrating 80% or better on each test has been tabulated. The percent showing mastery was obtained by counting the total number of
students who scored 80, 90, or 100% on the particular test and dividing that by the total number of students. This is recorded in Table 15.

<table>
<thead>
<tr>
<th>Pretest Mastery 80 % or More</th>
<th>Posttest Mastery 80% or More</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 15. Percent Showing Mastery Level on Pretest and Posttest

Results of Student Survey

Each student took an individual mathematics survey (Appendix A) before the program was introduced. This survey was to assess each student’s attitude about mathematics, if they felt comfortable asking questions in math class, if they studied mathematics outside of homework, and if they felt they could be successful in mathematics.

The same survey was given at the end of the seven lessons, after a time period of six weeks. The only difference was that the students were to answer the questions in regards to their experience using the Hands-On Equation System. Both surveys were read orally by the teacher to accommodate any students who might have difficulty with reading and/or understanding the questions.

In comparing the overall answers for the pre-survey and the post-survey, five responses remained the same, and ten responses were more positive after completion of the Level I Hands-On Equation Program. Students gave a more positive response related to enjoying math, feeling relaxed in math class, participation being considered acceptable
and find, students feeling more comfortable asking questions, and a more confident attitude in their knowledge and abilities in mathematics.

**Results of One-On-One Teacher Student Interviews**

Before the program was introduced, each student had a private interview with the teacher (Appendix B). These questions were to give the teacher some personal insight as to each student's thinking about mathematics. Some of the questions were based on the students' thinking about grades, how they felt they learned best, and what frightened them about mathematics.

At the end of the seven lessons, the same survey was to be given again. Since this was the first time I have used this program, I realized that the post interview questions needed to be adapted from the original survey. Using process validity, I adjusted the survey questions to more accurately report and interpret the results of using the *Hands-On Equation System*. This survey included changes in attitude about algebra, the lesson most enjoyed, what they would change about the program, if they wanted to continue with Levels II and III, and how they felt after reviewing all seven lessons on the videotape at the completion of the seven lessons (Appendix G).

An overall comparison between the pre-program interview and the post-program interview uncovered some very positive growth in attitude about mathematics and algebra. When the students were asked what they thought about algebra before they used the *Hands-On Equation Program*, the response was that algebra was hard and scary, and they didn't really know what algebra was. When asked how their attitude about algebra has changed as a result of learning the *Hands-On Equation System*, they responded that
algebra was fun and easier than they thought it would be, they felt smart because they
could solve the equations, and that now they know what algebra actually is.

When the students were asked which lesson or activity they enjoyed the most,
they all responded the ‘legal moves’. The name was catchy and the move was game-like.
This concept made the equations much easier to solve and it was fun to demonstrate the
‘legal moves’ for the group.

When asked what they would change about this system, everyone stated that
nothing needed to be changed in the program because they liked it the way it was. They
felt the program was great. They all wanted to continue with Levels II and III so they
could learn algebra, and they looked forward to it.

When all seven lessons were completed, the group watched the videotape from
the beginning to the end of Level I. When asked how they felt after viewing all the
lessons, some responded, “The equations in the beginning looked like they were going to
be hard and they now seem easy.”

Another student said, “Wow! I learned a lot, and I had fun.”

The overall feeling was that they felt smart because they could solve equations,
and they felt good about their success with this math program.

Summary

This chapter gave the results of the teacher action research using triangulation for
validity. The pretest showed that the students comprehended only one or two equations.
The posttest results showed all students achieved a mastery level of 80% or higher. Each
lesson was recorded including the method of instruction, the number of classes needed
for each lesson, the teacher’s observation about each student, and students' journal writing about the lessons. Finally, the results of the surveys and the teacher student interviews were reported.

Overview of Chapter 5

In chapter five, the data findings will be analyzed to determine how the research question was answered. Chapter five will contain my conclusions and reflections about using the Hands-On Equations System with seventh grade classified students. Chapter five will also include unexpected issues that may have impacted this study. I will include the results of an additional study, performed simultaneously, on two eighth grade classified mathematics students. They were not a part of this study, but the findings were equally important and will be reported. I will give my conclusions and reflection on these results. The chapter will conclude with a plan of action and rethinking on how the Hands-On Equation System could be used, or suggestions for modification, based on the outcome validity of this teacher action research. I will also reflect on the effects this teacher action research has had on me personally and professionally.
Chapter 5

Data Analysis and Conclusion

Introduction

The data gathered and reported in chapter four will be analyzed by examining the performance of the group, as well as the individual student’s performance. The purpose of this study was to look at the effects a manipulative hands-on approach to understanding variables and pre-algebra had on a small group of seventh grade students with learning differences. My research was focused on the use of a commercial program called the Hands-On Equation System. Would the students be able to set up and solve given equations? Would there be a change in attitude about mathematics as a result of using this system? How else would this system affect the students? This chapter will analyze the data and give my conclusions to answer those questions as well as others that have been raised through the research. The chapter will also contain my reflections on teacher action research and how this research has impacted my teaching. From the completed study come ideas and recommendations for other studies of a similar nature that can be enhanced from this experiment’s data and observations.

Interpretation of Data

Pretest

Results on the pretest (Appendix C) were basically as expected. Most students had correct responses for equations #1 and #2. Students were shocked when they received the pretest because they had not had algebraic equations before. They were assured that the score would not be used towards their report card grade, but as a
comparison to be used after they completed Level I of the *Hands-On Equation Program*. Students were told that they had unlimited time to do the pretest. The class mean score showed that the group averaged between one and two correct responses out of a possible ten. These correct responses were found exclusively in #1 and #2 only, which are common forms seen as early as first grade, using a box instead of a letter variable for the missing addend. Because students had no experience with the more complex algebraic equations correct responses were not expected.

Students A, D and E completed #1 and #2, and then rather quickly said they didn’t know how to do the rest. They appeared defeated and disappointed in only being able to attempt two problems. Students A and D only got #1 correct. Student E got #1 and #2 correct.

Students B and C at least looked over all ten equations and took advantage of the unlimited time available for the test. Student B attempted three equations and student C attempted four equations. Student B and C correctly solved only #1 and #2.

**Posttest.**

The results of the posttest (Appendix F), which was given three days after lesson seven was completed, were exciting. After observing the lesson by lesson progress the students had demonstrated, I anticipated good results. All students demonstrated at least 80% mastery. The class mean score for the posttest showed that the group averaged nine correct responses out of a possible ten. The pretest and the posttest were the same format but used different numbers. As the students compared the two tests, they could see their progress over the last six weeks and they were very impressed, as was I.
On the posttest, equations #8 and #10 could not be easily solved using the provided manipulative kit. The students were told at the start of the test that if they could not figure out a way to solve an equation using the manipulative kit, they might want to use the pictorial drawings learned in lesson seven. These two equations could be solved if the students recognized the need to combine like terms so as to have enough manipulative pieces available.

Students D and E solved #8 and #10 by using pictorial drawings. They got both equations correct and also achieved a score of 100% on the posttest. Student B was successful at combining like terms and was able to solve #8 and #10 using the manipulatives. Students A and C did the equations incorrectly because they just made the problem work with the manipulatives they had available. This was done by incorrectly changing $3(x + 2)$ to $2(x + 2)$. These two students ended up with a score of 80% on their posttest.

Student B was the only one to get problem #5 incorrect. This was due to his forgetting the final number in the equation, so even when the problem was checked, it seemed correct due to incorrectly writing the problem in the initial step. This student achieved a 90% score on the posttest.

Analysis of individual students.

The entire group increased their scores to the mastery level from the pretest to the posttest. All of the students were positively affected by the *Hands-On Equation System*. This part of the thesis will analyze each student individually.

Student A has just come out of a self-contained Language Learning Disability (LLD) classroom this past September. This student is extremely anxious and is having a
very difficult time making the transition to a mainstream placement. The educators are questioning this new placement, but the parents want the peer interaction for their child. As a result, student A is often frustrated, angry, and can be aggressive. Performance is frequently inconsistent and this student often appears mentally exhausted. Student A is thirteen years old and tested at 7% on the 2002 Terra Nova for the local composite math score.

Student A has good simple arithmetic skills, which has helped the student to be successful with this program. The student started to look forward to coming to class and would ask, "Are we going to do the fun stuff with the pieces?" I feel this student will need the concrete manipulatives to solve equations for an extended period of time. Each new lesson seemed to cause confusion for the student, but with coaching he could complete the worksheet on his own. Hopefully the student will eventually be able to progress to the pictorial drawings, and ultimately the abstract level of algebra. When student A was asked how he/she felt about the concepts learned in this program, he/she responded, "It felt good that I could do all that stuff. It was fun."

This program has proven to student A that he/she can be successful at something in school. With the difficult transition to the mainstream placement, this has been a needed boost for this student's self-esteem. The pretest score was 10%, and the posttest score was 80%. The student displayed great surprise and smiled as he/she realized the improvement made. The visual and kinesthetic aspects of this program made it ideal for this learning different student.

Student B has come out of an LLD classroom six months ago, and has made a good transition to the mainstream placement. This student has language processing
difficulties, and visual perception weaknesses. Student B is usually a consistent student though reminders to look at the numbers carefully are needed. The pretest score was 20%, and the posttest score was 90%. The one equation that was incorrect, was the result of not visually scanning the entire problem.

Student B is twelve years old and tested at 3% on the 2002 Terra Nova for the local composite math score. Student B felt the program was fun and easy from the start. Being able to view the exact lesson more than once really helped this student to process the new information. The student felt confident and did the worksheets without coaching. This student also enjoyed demonstrating and explaining the method used to solve the equations.

As I observed student B solving the equations, I knew this was whom this program was designed for. When the student was asked how he/she felt about the concepts learned in this program he/she responded, “I feel good about myself because when I first started this class I didn’t know if I could do algebra, but now I can.” This has been a very positive experience, especially for the student with learning differences. Student B is looking forward to continuing the Hands-On Equation Program.

Student C came out of the LLD classroom a year ago, and has had difficulty with the transition. Student C is academically capable but is having a hard time with the classroom and homework expectations. This student has ADD, without the support of medication, and only is focused when called upon to respond or read out loud. The student is emotionally fragile, which is sometimes demonstrated as anxiousness or depression.
Student C is frequently inconsistent in effort, attitude, and performance. The *Hands-On Equation System* made a major difference for this student. He/she was actively engaged and excited about the new lessons and worksheets. The student would volunteer to demonstrate and explain how to solve equations, which is out of normal character from my experience with the student. This program stimulated the student academically, while supplying the physical activity to keep the student focused. When a worksheet was completed, this student would return the individual kit with a smile upon his/her face, which is a rare sight indeed.

Student C is twelve years old and tested at 27% on the 2002 Terra Nova for the local composite math score. The pretest score was 20%, and the student was a bit surprised at his posttest score of 80%. This was a shock because the student always understood the lessons and did all equations correctly. Student C was able to recognize that he had been overconfident and disregarded checking the answers. The student wanted to rework the problems to discover his mistakes.

When the student was interviewed and asked what he/she thought about the program now that Level I was completed, the response was, “Wow, I learned a lot. I feel confident about learning algebra now. It was challenging, fun, and easy!” This student is looking forward to continuing with the program. To see this student looking forward to math class, and watching the student actively engaged in something academic was a treat. The *Hands-On Equations System* has been a wonderful encounter for student C.

Student D came out of the LLD classroom five months ago, and has made a good transition to the mainstream placement. This student has language-processing difficulties along with ADD and anxious qualities. This student is twelve years old and tested at
39% on the 2002 Terra Nova for the local composite math score. Based on the pre-
program survey and interview, it was clear that this student had math anxiety issues. This
student has struggled consistently with most of the mathematical concepts throughout the
previous school years. This student had very little self-esteem or confidence when it
came to learning algebra.

The Hands-On Equation System was wonderful for this student as well. The use
of the videotape to teach the lessons was very successful. This student would always ask
if the tape could be viewed again, sometimes up to four times. This allowed the student
more time to process the new lesson, while providing the student with the self-confidence
necessary for each new concept. For the first two lessons the student needed coaching for
part of the worksheet. Then the student started to trust the program and found he/she was
getting the correct answer consistently. This student said, “With this program I have
never had more fun in math. I like coming to math class,” which is a true success.

Observing this student, the confidence and self-esteem was obvious just by the
body language and the smile on the student’s face as the experiment continued. This shy
student wanted to demonstrate and explain the equations before the class. The post
program survey showed a dramatic positive change in attitude about math. When the
student was interviewed and asked what he/she thought about the program he/she
responded, “I thought all the stuff in the beginning was going to be hard, but now it
seems easy. I love this program!” This student was very excited about using the Hands-
On Equation System because he/she experienced success and understanding in using the
manipulatives.
Student E has always been in mainstream placement, with pullout resource room for major subjects. This student is oppositional and resistant to ordered procedures containing rules and steps. Student E is twelve years old and tested at 11% on the 2002 Terra Nova for the local composite math score. This student also has social issues with peers. From the survey and interview it is obvious this student has a poor attitude about mathematics. To say the least, this student is a challenge for any teacher.

Out of all the students, student E had a rocky start with this program. The student did not want to use or touch the manipulatives. (I felt the student regarded the manipulatives as items used by special education students, or for younger students.) The coach had to physically set up the equations on the balance for the student, and then encourage the student to use the set up to solve the equations. At first student E would only use a pencil point to move the pieces. By the third lesson the student was setting up the equations, but then would try to figure out the answer mentally. The student was told to guess and check but remained very resistant to following directions. By the end of lesson five, this student was finally getting the idea and began to trust the system. By the end of lesson six the student made a break through and wrote in the student journal, “I thought it was easy. I understand it and when all of this is done, I’ll know it all. I am surprised about the way I get the answers.”

Student E had a score of 20% on the pretest, and 100% on the posttest. Having made 100 on the posttest assured this student of his/her capability and a smile was seen on this student’s face! When the post survey and interview was given, this student showed positive feedback but in a way that displayed an ‘I don’t care’ attitude, showing this student’s oppositional trait. The student did like this program once he/she felt
confident in the procedure, and knew he/she could get the correct answers by following the steps.

This system was important to student E's mathematical growth, as well as the teacher's understanding of this student. Using a new system, which the student had no experience with, really allowed the teacher to discover just how fragile the student's understanding of basic mathematics really is. This student struggles with math concepts, but doesn't want anyone to know, so uses resistance and negative attitude to cover-up this deficiency. Student E keeps up a very tight guard and doesn’t let anyone in. The student stated he/she was interested in learning the other parts of the Hands-On Equation System.

The Research Question Answered Beyond All Expectations

The results of using the Hands-On Equations System have far exceeded my expectations. This quest began as I searched for a better way to teach students, who learn the subject of algebra differently. The problem is that algebraic equations are too abstract and most of my resource room students are concrete learners. Though I have used a variety of methods, including many of my own developed techniques, when it comes to making algebra concrete, I have never been fully satisfied with the results.

The Hands-On Equations System is an approach that provides concrete hands-on experiences to solve algebraic equations when students physically represent equations by using desktop game like pieces and 'legal moves'. Students are not bogged down with vocabulary and definitions, but learn intuitively that like terms can be combined without any discussion of this difficult abstract algebraic concept.
Borenson (1994) commented that "The Hands-On Equations Learning System is an algebraic learning environment which makes possible the Piagetian learning of algebraic concepts. Piagetian learning refers to learning without being taught, as discussed by Seymour Papert in his book Mindstorm (1980, p. 7). Some of our most powerful and lasting learning, such as learning to speak, are carried out via Piagetian learning" (Borenson, 1994, p. 10).

Borenson further stated that "early success with algebraic concepts provided students with a tremendous sense of mathematical power and self-confidence, bolsters students' mathematical interest, and provides students with an intuitive and concrete foundation for later algebraic work" (p. 2).

My initial expectation was hoping this system would indeed help my students to understand the concepts of algebra without the confusion and frustration most resource room students experience. What I discovered has been both academically enlightening and personally gratifying.

After taking a closer look at the results of the group and each individual student, the overwhelming conclusion is that this program has enabled all of the students to be successful at setting up and solving given algebraic equations. This is recorded by the dramatic difference in the pretest and posttest scores, as well as the successfully completed worksheets that accompanied each lesson.

This program has definitely changed the attitude that the students had about mathematics, especially algebra. This was observable in the students' confidence in using manipulatives to solve problems and communicate their ideas. They enjoyed demonstrating the equations and acting as the 'teacher'. Another place attitude change
was noticeable was in their flexibility to explore alternative methods to solve problems, and their willingness to persevere to see the equations through to completion. They felt positive about being able to solve equations with the manipulatives, and began to look forward to coming to math class. They displayed an interest and curiosity in solving the equations. While these students worked with the program, they were actively engaged in learning and participating. All of the students are looking forward to learning Levels II and III of the *Hands-On Equations System*.

As an instructor, I enjoyed teaching something new that none of the students had experienced before. I could approach instruction in the manner that I felt was best without dealing with baggage from prior instruction and techniques. The *Hands-On Equations System* was presented in orderly, simple steps that the students could understand and build upon. Dr. Borensen, who created this system, really paid attention to the construction of each lesson. This reminded me of the analogy made in chapter two, that like the construction of a tall building, a solid foundation is needed in teaching mathematics. These students discovered they could trust the seemingly simple lessons, and began to have a sense of mathematical power once they started to stack concepts on each other. Lodholz (1990) stated, “When attention was given to content, pacing, and exploring algebraic ideas, middle grade students were successful in algebra” (pp. 24-33). It was a wonderful sight to watch these students blossom with excitement as compared to the usual academic confusion and the feeling of academic defeat they so often experience.

Another unexpected bonus from this program was the self-motivation some of the students displayed in learning their addition, subtraction, and multiplication facts better
and faster. In Vaughn's book about teaching learning different students, he stated, "Interestingly, not all of their difficulties in mathematics relate to their knowledge of math, some relate to motivation" (2000, p.433). This program awakened a joy in learning, which fueled persistence and effort to learn the skills needed to manipulate and solve the equations.

Doing the simple mathematical operations provided additional insight as to each student's number sense. Students by seventh grade have learned to do simple mathematical calculations by rote, and can read numbers, so the instructor assumes they understand the value of the numbers. While these students were demonstrating how to solve the algebraic equations, it became clear who did and did not understand the numbers' inherent value. This knowledge will help to target those students who need additional instruction for basic understanding of numeration.

From a personal viewpoint, the most satisfying effect this program has made on these students is one of improved self-esteem, self-confidence, and the pride each student displayed as they successfully solved complex looking equations. For students who struggle daily with their individual learning differences, this program has been a wonderful experience. The *Hands-On Equations System* was successful not only in teaching fundamental concepts about algebra, but also by providing these students with a platform to build confidence in their ability to solve any equation.

**Modifications.**

The original time schedule allotted was very close to what I anticipated, but as far as individual lessons, more time was needed for lessons one and two than I had planned. This did not upset the overall time frame for this study because other lessons needed less
time than anticipated. I did plan extra time for the learning needs of these students. For a general math class, I feel two class periods would cover each lesson with students demonstrating the equations.

The other change made was to use the videotape to introduce the lessons. I had planned on introducing each lesson myself, and then I planned on using the videotape the next day as reinforcement of the lesson. I did present lesson one and the students had lots of questions. The next day I put on the videotape and they seemed more settled. What I slowly discovered was that after the students viewed the videotape, they would ask to watch it again. Of course I would respond positively to their request and the videotape was viewed again. As I observed the group watching the videotape, I could see them taking in the process step by step. What I realized was that many of the students with processing difficulty needed more time to think about what was said. As a result they would miss the next part. By watching the tape two or three times, these students could build on the knowledge they got from each viewing! I was amazed to hear them say, “Okay, now I get it!” All I had to do was replay the videotape.

This insight has caused me to rethink my teaching strategies for students with processing difficult. I have always presented a new concept in many ways to hopefully find a way for all my students to understand what is being taught. Now I am wondering if some of the students just need the same presentation repeated, with the same actions and words so they can build upon the steps they got the first time and then process what was originally missed. Maybe it isn’t how I present a concept, but the fact that some students just need more time to process the information before they can add onto it. Perhaps the use of a video camera would be helpful in doing an instant replay of a
teacher's presentation. This videotape could even be taken home for viewing before
doing homework.

I shared this idea of identical replication of teaching with the head of the special
services department. She had a look of reflection at first exposure to this new way of
looking at teaching some of our special needs students. She felt this was an area to
research and with which to experiment. This shows that good research never really ends,
but spirals off and continues into new directions.

Research and literature.

As this study was being conducted, I would often be reminded of the literary
research I had completed for chapter two of this teacher action research. It is
unquestioned that the study of mathematics is important to one's future problem solving
ability in life. Algebra has been identified as the "gatekeeper" not only for math, but also
for science and other higher learning. Algebra is so central to mathematics that if
students do not learn it, and learn it properly, their future options can be severely limited
(Willis, 1993). In New Jersey all students must pass the HSPT, which includes algebra,
in order to receive a high school diploma. Also, most students who want to go to college
must take an entrance exam, which includes algebra.

The Little Falls Middle School introduces pre-algebra in the seventh grade. I feel
students should be introduced to algebraic equations at an earlier age so they have a
comfort level when they see equations. The NCTM suggests, "By integrating informal
algebraic experiences throughout the K-8 curriculum, students will develop confidence in
using algebra" (NCTM, 1989, p. 104). The Hands-On Equations System can be used as a
supplemental program in conjunction with the regular math textbook beginning as early
as the fourth grade. Then, each consecutive school year, another level of the program can be taught. By eighth grade these students should have a solid foundation of algebraic concepts upon which to build.

Many researchers indicated that students’ learning evolves through stages and that the use of concrete manipulatives is important to bridge the gap between concrete and abstract reasoning. Kieran (1991) stated, “To help students construct meaningful mathematics internally for themselves, physical materials were necessary” (p. 224). In the *Hands-On Equation System*, once the student has concrete understanding of the concept, they will progress to a pictorial stage to span the gap before reaching an abstract level.

Herbet (1985) talked about the use of game-like manipulatives, “Many younger students can learn some of the basic abstract concepts of algebra through games, and using manipulatives. Manipulatives motivate students: manipulatives stimulate students to think mathematically” (p. 4). The students in this research study loved the game-like ‘legal moves’ and looked forward to using this program in math class.

As I looked back over the claims that Dr. Borenson made about his system, I came across this statement. Borenson (1998) claimed:

Students are impressed with their ability to solve algebraic linear equations in a game-like manner. The ‘legal moves’ provide students with a sound, intuitive understanding of fundamental algebraic properties without realizing they are learning them. The early acquisition of these concepts maybe an essential step in helping to raise the level of mathematics education in the United States. Students will experience success and enjoyment in solving what looks like sophisticated
equations. This serves to enhance self-esteem and interest in mathematics. The *Hands-On Equation System* program is for students of average ability, gifted students, and for the student with learning disabilities. The visual and kinesthetic aspects of this program make it especially ideal for use with learning disabled students at any time from the fifth grade on (p. 4).

After teaching Level I of this program and rereading the ideas put forward by Dr. Borenson, it was clear in my opinion that the claims were accurate. At the beginning of this research I was skeptical of observing these outcomes, but I am delighted to say, with my seventh grade resource room students, that this program does just what Borenson assured.

**Unexpected situation.**

Many school districts in the state of New Jersey were under construction during this school year, and Little Falls Middle School was one of these schools. As a result, I did not have a permanent teaching space for my pullout resource room students. This math group was in a different location everyday of the week. This caused some uncertainty about where the class was to meet each day, as well as not being able to have the materials set-up prior to class. Additionally, four times a week we had to share the teaching space with another small group. This had a definite negative impact upon the students and the instructor. At times these students would be heard saying, “The other kids (meaning the general education students) are lucky because they have a room to be in. We have to travel all over the school.”

The lack of facilities caused a loss of instructional time due to setting up the space for class, along with the transportation of the materials from room to room. I feel if there
was a permanent location for this class, the comfort level and routine would have
provided more time for the task, and the students would have been even more focused on
the program.

The one positive aspect of not having a permanent location for this class was that
I had the *Hand's-On Equations System* with manipulatives and game-like moves to keep
the students actively engaged. They looked forward to math class, and I was amazed
how focused and involved these students were under these conditions. I consider myself
very fortunate to have had this new program to research and try out.

**Reflections on Teacher Action Research**

**The importance of teacher action research.**

Doing this teacher action research has changed not only my self-confidence as a
teaching professional, but this experience has empowered every aspect of my life! As I
reflect back in time to when I first thought about taking graduate courses to go beyond
my BA degree, there were uncertain feelings of not being able to handle going back to
school. I had graduated from college twenty-seven years ago, raised my children, and
returned to teaching eleven years ago. Hearing about the Regional Training Center
(RTC) graduate courses, I decided, along with two friends, to try a course. Needless to
say, I loved the classes and continued to further my education.

I enjoyed learning useful information that would improve my teaching techniques,
working cooperatively with other colleagues, and being viewed as a professional. This
was an up-lifting experience, and I began to teach with a renewed energy and enthusiasm.
I became reacquainted with the local university library, and actively started reading
professional journals. I began to feel like an authority on the topics I had studied from the RTC courses, and self-confidence was growing. I knew I wanted to continue educating myself and decided to earn a Master of Education degree.

When it was time to begin the teacher action research part of the Master's program, I was once again filled with self-doubt and anxiety. After a full week of instruction, guidance, and encouragement from my advisor, I began to embark on this daunting task. Through out this process, my teacher action research study group would meet and we would encourage each other. Support was also received from my colleagues, and my family who often got me over the temporary hurdles.

Chapter one of the teacher action research was self-reflective and made me stop and take a look at where I started my teaching career, and where I was going in my career, gaining unexpected insight into myself. This chapter also made me pin point my research question. This was harder than expected, but very necessary for the research.

Chapter two was the literary research and I found out what the experts had to say about my topic. From doing the literary research, I learned so much that at times I thought my brain could not hold any more information, much like many of my students feel daily. According to Glanz (1998), teacher action research is a more disciplined systematic approach to study a given educational problem. I feel the one step missing when teachers want to promote a change in the curriculum is the literary research. Often teachers draw conclusions based upon observations and intuitive feelings. For credibility one must do research into what is said by other experts. This research needs to avoid bias and also helps to separate facts from opinions.
Chapter three was exciting as I planned out just how I was going to do my research. This was an area of strength because planning the delivery and timing of lessons is what I do everyday as a teacher. The part I had to really think about was how I was going to collect the data necessary to analysis the results of this study. I decided to use a variety of ways to collect data to provide a more complete picture, and to ensure that the results had validity. The collection tools used were based on triangulation. On the topic of triangulation, Marshall and Rossman explain, “Triangulation is the act of bringing more than one source of data to bear on a single point. Designing a study in which multiple cases are used, multiple informants or more than one data gathering technique can greatly strengthen the study’s usefulness for other settings” (Glanz, 1998, p. 41). This method of triangulation has been one of the useful techniques learned from this teacher action research, and will certainly use in the future.

Chapter four and five presented the data collected based on triangulation, and my conclusions. Teacher action research is cyclical in that the conclusion is not the end of the study, but only the beginning of action and new ideas to explore. I completely agree with Glanz who noted teacher action research is systematic, intentional, and ordered. It empowers those who participate in the process, creates a more positive school climate, promotes reflection and self-assessment, enhances decision making, and instills a commitment to continuous improvement (Glanz, 1998). I am looking forward to using teacher action research in my future teaching career. I feel this will afford me the opportunity and tools necessary to accomplish professional satisfaction and boost the achievements possible by the profession of teaching. This truly has been an empowering, enlightening, and fulfilling experience.
Validity.

This research was qualitative and internal. Due to my role as the teacher in this research, my daily decisions affected the data. The results of this study are particular to the students involved in the study and can not be generalized to other situations. Throughout the study, I engaged in process validity through adjustments made in the schedule of instruction to fit the setting and flow of the day. The decision to modify, by adjusting the post interview questions, was influenced by students' input and enthusiasm after experiencing the program. The study has dialogic validity due to the support of the administration and staff. Teachers and the principal would often stop to discuss my progress with this research. My teacher action collaborative group, and my family, supported me by sharing ideas and knowledge. This research has outcome validity, as the completion of this project will not be a means to an end, but an ongoing learning experience. Most importantly, the research has catalytic validity. I have been deeply impacted by this research. This program has had a positive affect on my students and I hope to extend this opportunity to the general school population in the future. This study also contains outcome validity because it has raised many questions about possible uses and effects the program could have on other students. (Anderson, Herr & Nihlen, 1994).

I also consider the results of this research valid based on the triangulation of the data and the support of the literature. The students' input from surveys, interviews, and journal writing, provided student voice and showed positive changes as a result of experiencing the program. Teacher observation recorded visible changes in performance, self-esteem, and attitude about algebra. Student assessment scores, which consisted of
existing records and the scores obtained during the study, provided measurable evidence
of the growth obtained by each student.

**Impact of the Research on My Teaching**

**Putting the plan into action.**

As a result of this teacher action research, I am making some meaningful changes
in my instructional strategies. As far as the group this study used, the results from
implementing this manipulative approach are undisputed. The students were able to
solve complex looking algebraic equations, and they could explain their plan. The
students’ attitude about algebra was positively impacted as they began to look forward to
math class. They now experienced success in this class. The use of manipulatives also
kept all the students actively engaged and focused on the task at hand. From now on, I
will be using the *Hands-On Equations system* in teaching mathematics. I love that I will
be able to use this system with any of the grade levels I teach since this program has been
designed for fourth grade and above.

I put this action research into immediate motion by using the system with eighth
graders with whom I teach math in the resource room. This gave me a new perspective
of the program because this group had been taught how to solve algebraic equations last
year and I was the instructor. This was the group that made me actively look for a better
way to teach this subject because I knew I did not do enough to make it come alive. As a
result they didn’t really understand the process and had a hard time remembering what to
do, and when to do it.
This group of students was able to verbalize the connection to what they learned last year. As they used the Hands-On Equations System the process of solving the equations became clearer. They began to understand the concept of the equal sign and what it means. This amazed me because I thought they already understood that! The concept of variable also became clearer, which is a very difficult concept for student to get. They really enjoy using the manipulatives, and I notice they are more willing to switch to the pictorial representation of the equations than the seventh graders. These students are already into Level II of the program and want to complete the program. They want to make the connections to what they learned last year. I am hopeful that by the end of the program they will be able to make that connection on their own. Once again, this shows that good research is always continuing.

The bigger picture.

As I conducted this study and shared the on-going results with the principal and other math teachers, the idea of using this program school-wide became a definite possibility. The principal asked me to keep him posted as to the results of the research. He said he would like me to do a presentation for the math teachers in the entire middle school so they would be familiar with the program and my results. I told the principal I would be happy to present the program to my colleagues as well as to the students in all grade levels.

I was asked to do a demonstration of the Hands-On Equations System for an advance math class in the seventh grade. They were just starting to solve algebraic equations and their math teacher felt the class might get a better understanding of the concepts if they saw this technique. I found it interesting that this advanced group of
mathematics learners could really focus in on the concepts being taught. These students did not have the individual kits to use. I only used the large demonstration balance due to the time available. Many of these students seemed to understand equations and variables better after this condensed presentation of the seven lessons. They were able to move from one lesson to the next very quickly and wanted to continue the hands on learning further.

There were also social benefits from this macro presentation of the *Hands-On Equations System* to the advance seventh grade math class. As a special education teacher I am concerned with the self-esteem of the classified students, and the way the general population of students perceive them. As I demonstrated the lessons to the advance math group, I told them at my conclusion that my resource room students had already mastered this system. They were visibly bewildered. This gave me the opportunity to explain that resource room students learn differently but they learn the same things the mainstream students are learning. Many seemed surprised by this information, yet others understood. Examples like this help to reduce the stigmas attached to classified students and hopefully one day be erased.

When I reported to my resource room group what had happened, they were concerned at first that their teacher was teaching in the advanced math class. How could that be? I told them that the other group was impressed that the resource room students were doing algebraic equations too. I gave my students the same talk about how being in a resource room setting did not mean the inability to fully learn all the concepts. I think they were surprised they were actually ahead of the advanced math group, and that these manipulatives were not just for special education. I could sense a feeling of self-worth in
my students. They want to be like everyone else, and maybe it was okay that they learn differently. In my opinion, teaching is about the growth of the whole person not just academic scores.

Summary

This teacher action research set out to examine the effects a hands-on manipulative program would have on a group of seventh grade resource room math students. The program is called the *Hands-On Equations System*, and I have found the claims made by this program to be true for this particular group of students. They can physically set-up and solve given algebraic equations, as well as explain the process used in logical progression. This program has changed math attitudes for the better, as these students have been successful in solving complex looking equations. These students have displayed improved self-confidence and self-esteem in math class. This program has found a permanent place as a part of my math curriculum in the future. The middle school principal is interested in the general population of students being exposed to the exceptional results that my students have experienced.

Doing this teacher action research has been mostly intense, but ultimately has enlightened and empowered me as a professional teacher. I have undoubtedly grown personally and professionally, and I will take the ideas and concepts from this process forward with me in my teaching strategies and my life.
Bibliography


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(APPENDIX A) STUDENT SURVEY

MATHEMATICS SURVEY

1) Walking to math class I feel:

- Happy, pretty good
- Normal, just okay
- Nervous, uneasy

2) I have always enjoyed mathematics:

- Yes!
- It’s okay
- No, it makes me nervous

3) When I am in math class, I usually feel:

- At ease and relaxed
- A little on edge
- Not relaxed, stressed

4) I like to participate in math class:

- Yes, it’s fun!
- Sometimes
- No, not at all
5) When something is unclear, I am comfortable asking questions in class.

I often feel this way                       I never feel this way

1  2  3  4  5

6) When I am taking a math test, I usually feel:

Very nervous & uneasy                      Pretty good, confident

1  2  3  4  5

7) I am comfortable asking a question in class if I don’t understand something:

I have no trouble asking questions         I’d NEVER raise my hand in class

1  2  3  4  5

8) I study mathematics outside of class and outside of homework:

I only do what I HAVE to do                I do study mathematics and use it in my daily life

1  2  3  4  5

9) Math class must be fun in order for me to learn:

MUST be fun                             Fun would be great!          Fun is not needed
for me to learn!                         For me to learn!
10) I feel I am good at solving math problems:
   strongly agree  agree  disagree  strongly disagree

11) Mathematics can be interesting and exciting:
   strongly agree  agree  disagree  strongly disagree

12) I feel self-conscious when I’m with people who are very good in math
   strongly agree  agree  disagree  strongly disagree

13) I feel I can be successful in mathematics:
   strongly agree  agree  disagree  strongly disagree

14) It is important that I like the teacher in order for me to learn:
   strongly agree  agree  disagree  strongly disagree

15) Math homework is important so I can practice math skills:
   strongly agree  agree  disagree  strongly disagree

16) People learn things in different ways:
   strongly agree  agree  disagree  strongly disagree
(APPENDIX B) TEACHER / STUDENT INTERVIEW

INTERVIEW QUESTIONS

1) What do you think teachers should do to help students remain confident while learning math concepts?

2) Are grades important to you? Why?

3) What is the most frightening aspect of mathematics to you?

4) How do you think you learn best?

5) What effects your attitude towards learning?
(APPENDIX C) PRETEST

PRETEST

Find the value of x for each:

1) \( x + 6 = 9 \)  \quad x =

2) \( 10 = 8 + x \)  \quad x =

3) \( 2x + 4 = 10 \)  \quad x =

4) \( 3x = x + 2 \)  \quad x =

5) \( 13 + x + x = x + 3 + 3x \)  \quad x =

6) \( x + 3x + 12 = 2x + 20 \)  \quad x =

7) \( 6 + 2x + 4 - x = 2x + 3 \)  \quad x =

8) \( 2(x + 5) = 4x + 2 \)  \quad x =

9) \( 2(3x + 1) = 3x + 20 \)  \quad x =

10) \( 3(x + 1) = 10 + 2x \)  \quad x =
The Hands-On Equations® Learning System

Level I

Objective: By the end of the seventh lesson, elementary school students (third grade and up) will be able to physically set up and solve such equations as

\[ 2x + x + x + 2 = 2x + 10 \]

and

\[ 2(x + 4) + x = x + 16. \]

Materials Needed Per Student:

- Eight blue pawns
- Two red cubes, numbered 0-5
- Two red cubes, numbered 5-10
- A laminated balance scale
Materials Needed by the Teacher:

- Pawns and cubes as above, but larger;
- A stationary physical scale

**Lesson #4**

In the first lesson, the teacher displays on the physical scale in front of the classroom, problems such as

\[ \triangle \quad \square \]

and

\[ \triangle \triangle \quad \square \]

**FOR HOME USE:**

Please display these problems on the enclosed laminated balance scale.

Once students grasp the concept that both sides of the scale must have the same value for the scale to balance, they see that the pawn in the first problem is worth 5, and that in the second problem it is worth 4. Students can then be presented with other “physical equations” which they are to solve by trial and error methods.

**Ex.** \[ \triangle \triangle \quad \square \quad \square \triangle \]

The students see that “1” does not work since both sides are not equal. “2” does not work, etc. “6” does work since the left side is now 14 and so is the right side. The students are informed that the pawn has a special name, “x,” and that there is a special way of writing the answer:

\[ x = 6 \text{, check: } 14 = 14. \]

The students are given Student Kits so that they can set up the worksheet problems at their desks. (On the student setup, it is helpful if the number-cubes are facing upward, i.e., facing the ceiling, so that the teacher can easily see if the students have the correct setup.)
Comments on Lesson #1

In this lesson, students begin learning about equations, variables, and unknowns on both sides of a setup—but they do so intuitively, through Piagetian learning. Indeed, the word "variable," which can even scare some adults, is not used at all. Important algebraic concepts are nonetheless acquired in a very natural way as the students work with the materials.

Lesson #2

Students are reminded that the pawn has a special name, "x." Therefore, the problem

\[2x + x = x + 8\]

really calls for placing "two x's" and "one x" on the left side of the scale, and "an x" and an "8-cube" on the right side:

\[
\begin{array}{c}
\text{\textbullet \textbullet \textbullet} \\
\text{\textbullet} \\
\end{array}
\text{\textbullet } \text{\textbullet }
\]

The students are given their Student Kits so that they can set up the problem at their desks. Then, they can solve by trial and error methods as in Lesson #1. The answer is:

\[x = 4, \text{ check: } 12 \leq 12.\]

Other examples which the teacher may use in this lesson include

\[3x + 1 = x + 7\]

and

\[4x = 3x + 5.\]

Comment on Lesson #2

It is a tremendous credit to the power of this system that young students can interpret and make sense of the above problems after only two lessons!

Lesson #3

The teacher begins by posing to the class a problem such as

\[4x + 2 = 3x + 9.\]

The students set up this problem at their desks and attempt to solve it. Because this problem may stymie many students, it offers an excellent opportunity for the teacher to say:
"Would you like to learn an easier way of getting the answer than by using trial and error?"

The teacher can now proceed to see if the students "buy" the idea (which the teacher now physically demonstrates), that if one pawn is removed simultaneously from each side of the balanced setup,

that the scale will still balance. Such a move, which leaves a balanced system in balance, is called a "legal move." (To confirm that the students do in fact understand this key concept, the teacher should then attempt to remove two blue pawns from the left side and one blue pawn from the right side. The students should see that this move is not legal.)

By carrying out the above legal move two more times, the setup now shows

from which the students can easily see that \( x = 7 \). The teacher can now say to the class:

"Let's see if our method has worked. Let's physically set up the original problem one more time to see if \( x = 7 \) makes both sides balance."

A student can come up to the front of the room and, after physically* resetting the original equation, verify that if each pawn is worth 7, the system

balances since \( 30 \equiv 30 \). So the answer, \( x = 7 \), is correct.

*In Hands-On Equations, the check is always carried out in the original physical setup, not in the original abstract equation. The physical setup is the concrete meaning of the abstract equation.
The teacher can now assign other similar problems for students to set up and solve at their seats, using legal moves if they wish:

\[
\begin{align*}
\text{Ex. } 5x + 2 &= 2x + 14 \\
\text{Ex. } 2x + x + 4 &= 4x + 1
\end{align*}
\]

**Lesson #4**

In this lesson, students learn that subtracting the same number-cube value from each side of a balanced setup leaves the setup in balance.

\[
\begin{array}{c|c}
\triangle & \square \\
\hline
\end{array}
\]

Given the above setup, students can subtract a 4-value from the cubes on each side,

\[
\begin{array}{c|c}
\triangle & \square \\
\hline
\end{array}
\]

thus leaving

\[
\begin{array}{c|c}
\triangle & \square \\
\hline
\end{array}
\]

Sometimes this process is more clearly illustrated if the 10-cube is first replaced by a 4-cube and a 6-cube,

\[
\begin{array}{c|c}
\triangle & \square \\
\hline
\end{array}
\]

before removing the 4-value from each side.

So far, then, the students have learned two legal moves: that they may subtract the same number of blue pawns, or \( x \)'s, from each side of a balanced setup, or that they may subtract the same value from the cubes on each side. The following example enables the students to perform both of these legal moves. A possible solution sequence is shown.
Ex. $4x + 5 = 2x + 13$

So, $x = 4$. The check in the initial physical setup reveals that $21 \neq 21$.

Other problems which can be given in this lesson include

$2x + x + x + 2 = 2x + 10$

and

$x + 3x + 3 = x + 18$.

**Lesson:**

In this lesson, students take away pawns as part of the setup process:

$5x - 3x + 2 = x + 5$

From this, the original physical setup, students can now proceed to use legal moves. After removing one blue pawn from each side, we see that $x = 3$. The check, in the above setup, shows that $8 \neq 8$. 
Other examples which the teacher can assign in this lesson include
\[ 2x + x - x + 1 = x + 9 \]
and
\[ 4 + 3x - 2x + x = x + 5. \]

**Lesson #6**

In this lesson, students learn to solve such equations as
\[ 2(x + 3) = x + 8. \]

They learn that the "2" outside the parenthesis means that what is inside the parenthesis, the "x + 3," is to be doubled. Hence, the student setup for this problem is

![Diagram](image)

By having the students display the doubled portion in two rows on the mat, the teacher can easily check that the correct elements have been doubled. (In the teacher setup, the pawns and cubes are placed next to each other,

![Diagram](image)

so that they are visible to the class.) By subtracting one x from each side, we see that \( x = 2 \); check \( 10 \leq 10 \).

Other examples the teacher can give in this lesson include
\[ 2(2x + 1) = 18 \]
and
\[ 2(x + 4) + x = x + 16. \]

**Comments on Lesson #6**

It is fascinating to see that elementary school students, when instructed in this manner, have little difficulty in working with a multiple of a parenthetical expression. Occasionally, they "discover" the distributive law on their own and double each element inside the parenthesis in sequence.

**Lesson #7**

In this lesson, students transfer their concrete, hands-on experience in solving algebraic linear equations to a pictorial system involving only pencil and paper. The technique is illustrated in the two examples below.
Ex. $4x + 3 = 3x + 9$

Solution:

$x \times x \times \boxed{3} \times \boxed{6}$

So, $x = 6$. Check: $27 \neq 27$.

Ex. $2(x + 4) + x = x + 16$

Solution:

$x \boxed{4} \times \boxed{8}$

So, $x = 4$. Check: $20 \neq 20$.

Thus in this pictorial notation, the student draws a scale, and on it, places written "x's" (instead of pawns), places written boxed-numerals (instead of number-cubes), and crosses off or places arrows above anything that is to be taken away. Once the answer is obtained, the student goes back to the initial pictorial setup (redrawn, for clarity) in order to carry out the check.

Note: Some students prefer to use pictures of the pawns, rather than written "x's," to represent the physical pawns.

Ex. $4x + 3 = 3x + 9$

Solution:

$\boxed{3} \boxed{6}$

So, $x = 6$. Check: $27 \neq 27$.

This pictorial notation more closely resembles the actual physical setup and is therefore easier for some younger students and some students with learning disabilities to understand. This notation is perfectly acceptable. (The picture of the blue pawn is shaded in, to distinguish it, in Level II, from the picture of the white pawn which is not shaded in.)
HANDS-ON EQUATIONS®

Lesson #5

Classwork Sheet

Name:__________

Grade:__________

Use Your Hands-On Equations Kit to solve:

New Work

1. \[4x - 2x + 4 = x + 10\]  \[x=\]  Check:_______

2. \[x + 3x - x = 30\]  \[x=\]  Check:_______

3. \[8 + 2x + 3 - x = 2x + 1\]  \[x=\]  Check:_______

4. \[x + 2x - x + 7 = 4x - x + 2\]  \[x=\]  Check:_______

Previous Work

5. \[\text{V V 3, 10 5}\]  \[x=\]  Check:_______

6. \[\text{V 4 V, V 8}\]  \[x=\]  Check:_______

7. \[4x = 2x + 4\]  \[x=\]  Check:_______

8. \[x + 3 + 2x = x + 3\]  \[x=\]  Check:_______

9. \[2x + x + 4 + 2x = x + 20\]  \[x=\]  Check:_______

10. \[2x + 8 + x = 2x + 15\]  \[x=\]  Check:_______
(APPENDIX F) POSTTEST

POSTTEST

Find the value of $X$ for each:

1) $x + 8 = 12$  \hspace{1cm} x =

2) $7 = 4 + x$  \hspace{1cm} x =

3) $2x + 6 = 14$  \hspace{1cm} x =

4) $3x = x + 10$  \hspace{1cm} x =

5) $x + x + 12 = x + 5x + 8$  \hspace{1cm} x =

6) $5 + 3x + x = 3x + 9$  \hspace{1cm} x =

7) $3 + 4x - x + 11 = 5x + 6$  \hspace{1cm} x =

8) $3(x + 2) = 4 + 4x$  \hspace{1cm} x =

9) $2(2x + 5) = 3x + 13$  \hspace{1cm} x =

10) $3(x + 1) = 5 + x$  \hspace{1cm} x =
(APPENDIX G) TEACHER/STUDENT POST PROGRAM INTERVIEW

Interview Questions

1. How did you feel about algebra prior to learning the *Hands-On Equation System*?

2. Describe how your attitude and perception of algebra have changed as a result of this program?

3. Describe the lesson or activity that you enjoyed the most.

4. What would you change about the *Hands-On Equation System*? Why?

5. What skills did you acquire as we went through the program?

6. Where might you use those skills?

7. Would you like to continue onto level II and III of the *Hands-On Equation System*?

8. After viewing the entire videotape on all seven lessons, how did you feel? What did you think?